LV. *An Attempt to explain some of the principal Phænomena of Electricity, by Means of an elastic Fluid: By the Honourable* Henry Cavendish, *F. R. S.*

Read Dec. 19, 1771, and Jan. 9, 1772. SINCE I first wrote the following paper, I find that this way of accounting for the phænomena of electricity, is not new. Æpinus, in his Tentamen Theoriæ electricitatis & magnetismi, has made use of the same, or nearly the same hypothesis that I have; and the conclusions he draws from it, agree nearly with mine, as far as he goes. However, as I have carried the theory much farther than he has done, and have considered the subject in a different, and, I flatter myself, in a more accurate manner, I hope the Society will not think this paper unworthy their acceptance.

The method I propose to follow is, first, to lay down the hypothesis; next, to examine by strict mathematical reasoning, or at least, as strict reasoning as the nature of the subject will admit of, what consequences will flow from thence; and lastly, to examine how far these consequences agree with such experiments as have yet been made on this subject. In a future paper, I intend to give the result of some experiments I am making, with intent to examine still further the truth of this hypothesis, and to find out the law of the electric attraction and repulsion.

Fig. 1.

A B D E
b d e

Fig. 2.

β
B
b
P
C
e
E
ε
d
D
δ

Fig. 3.

b
d
B
D
A
c
C

·P

A ——————————— a
B ------------------- b
C ------------------- c
D ——————————— d

·S

E ——————————— e
F ------------------- f

g
G ------------------- g
H ——————————— h

Fig. 4.

Fig. 5.

B

Fig. 6.

B

M
G · H
N

Fig. 2.

Fig. 7.

Fig. 5.

Fig. 6.

Fig. 4.

Fig. 7.

M

G

A

P

p

m

H

D

n

N

Fig. 8.

A

C

D

F

B

G

F

D

A

D

G

E

C

F

B

Fig. 10.

a

A

B

b

D
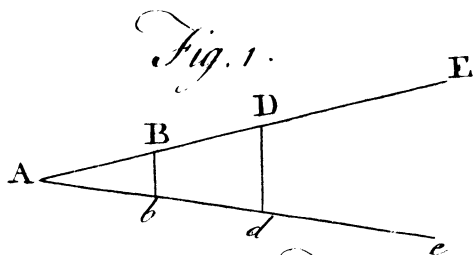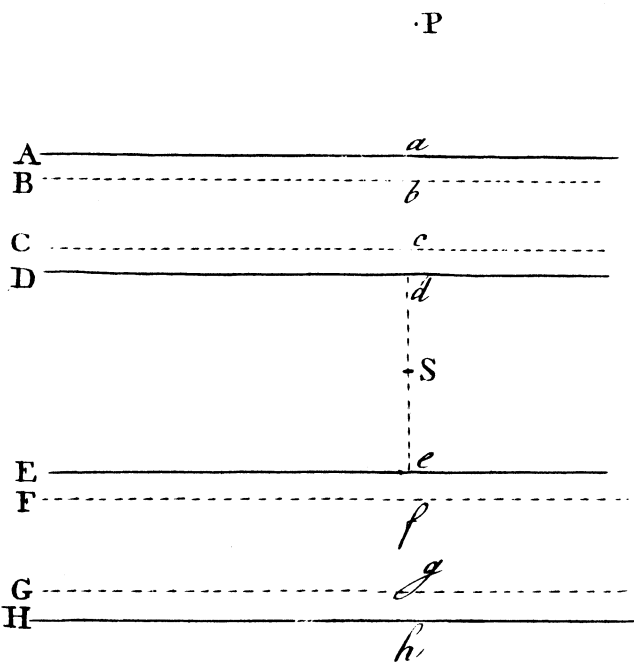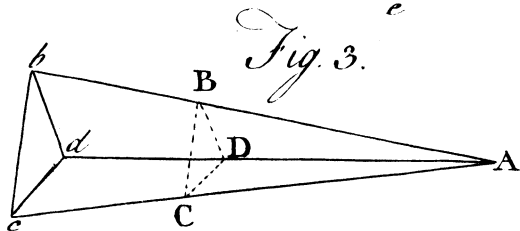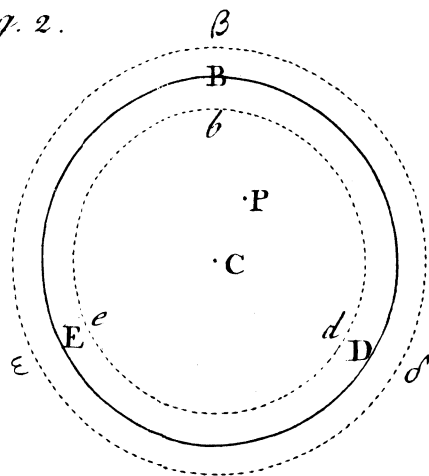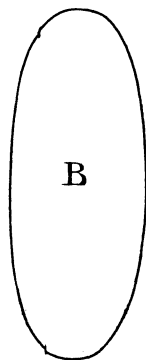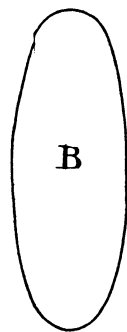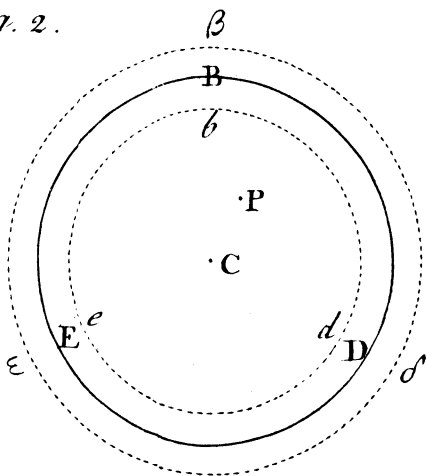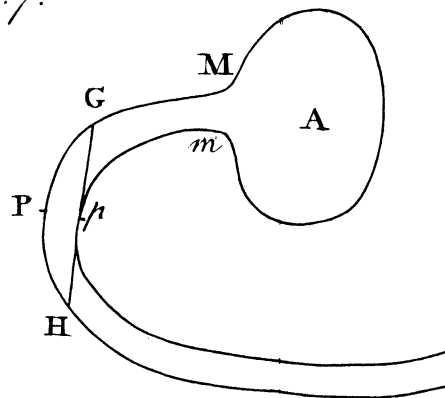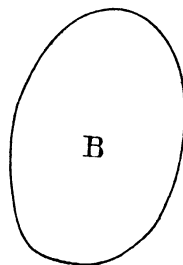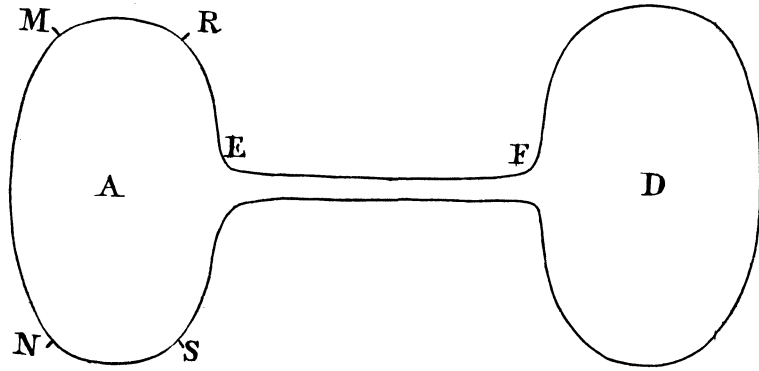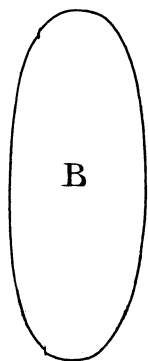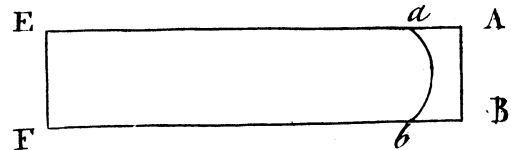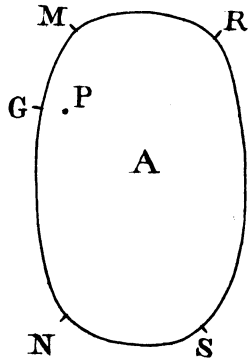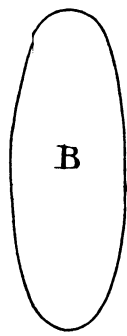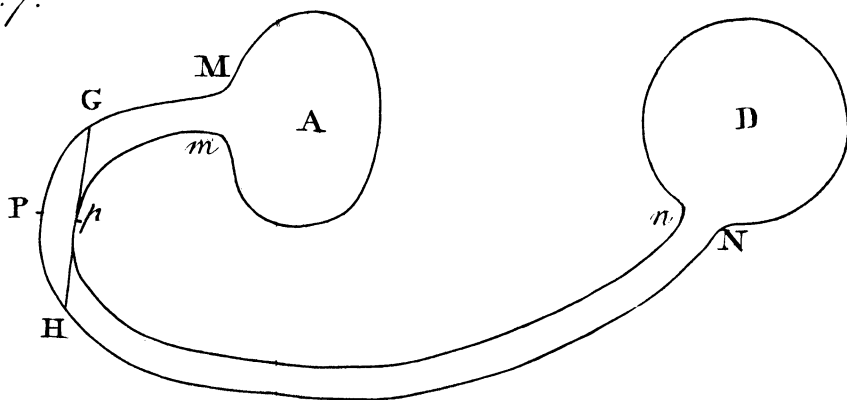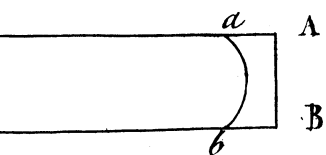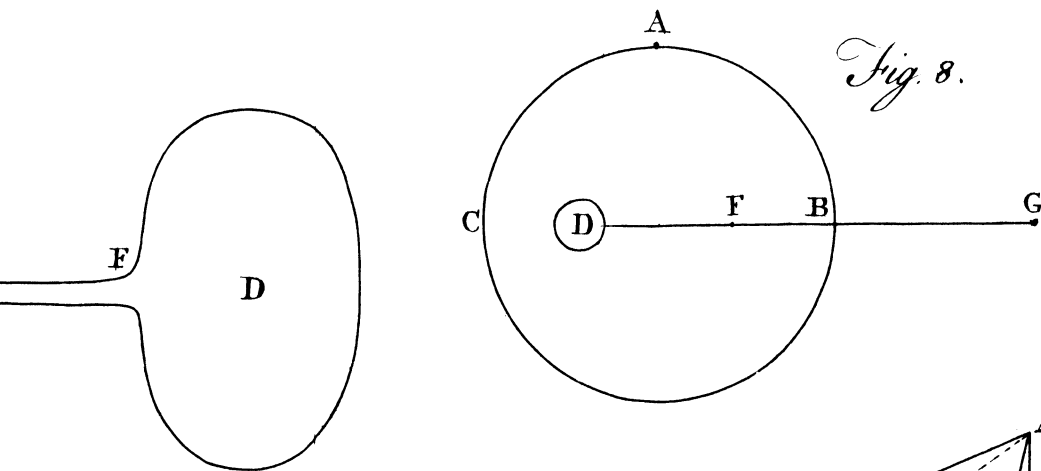
Fig. 9.
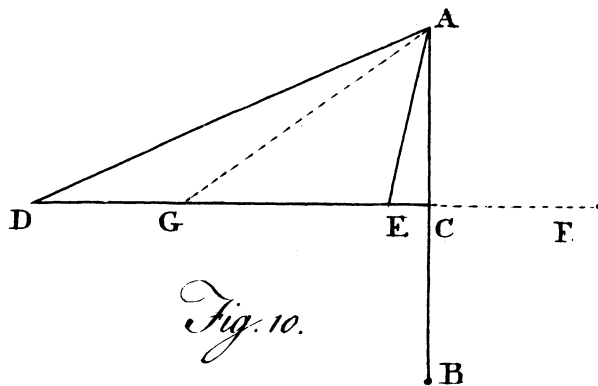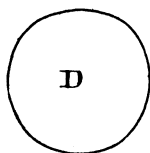
HYPOTHESIS.

THERE is a fubftance, which I call the electric fluid, the particles of which repel each other and attract the particles of all other matter, with a force inverfely as fome lefs power of the diftance than the cube: the particles of all other matter alfo, repel each other, and attract thofe of the electric fluid, with a force varying according to the fame power of the diftances. Or, to exprefs it more concifely, if you look upon the electric fluid as matter of a contrary kind to other matter, the particles of all matter, both thofe of the electric fluid and of other matter, repel particles of the fame kind, and attract thofe of a contrary kind, with a force inverfely as fome lefs power of the diftance than the cube.

For the future, I would be underftood never to comprehend the electric fluid under the word matter, but only fome other fort of matter.

It is indifferent whether you fuppofe all forts of matter to be indued in an equal degree with the foregoing attraction and repulfion, or whether you fuppofe fome forts to be indued with it in a greater degree than others; but it is likely that the electric fluid is indued with this property in a much greater degree than other matter; for in all probability the weight of the electric fluid in any body bears but a very fmall proportion to the weight of the matter; but yet the force with which the electric fluid therein attracts any particle of matter muft be equal to the force with which the matter therein

repels

repels that particle; otherwife the body would appear electrical, as will be fhewn hereafter.

To explain this hypothefis more fully, fuppofe that 1 grain of electric fluid attracts a particle of matter, at a given diftance, with as much force as n grains of any matter, lead for inftance, repel it: then will 1 grain of electric fluid repel a particle of electric fluid with as much force as n grains of lead attract it; and 1 grain of electric fluid will repel 1 grain of electric fluid with as much force as n grains of lead repel n grains of lead.

All bodies in their natural ftate, with regard to electricity, contain fuch a quantity of electric fluid interfperfed between their particles, that the attraction of the electric fluid in any fmall part of the body on a given particle of matter fhall be equal to the repulfion of the matter in the fame fmall part on the fame particle. A body in this ftate I call faturated with electric fluid : if the body contains more than this quantity of electric fluid, I call it overcharged : if lefs, I call it undercharged. This is the hypothefis; I now proceed to examine the confequences which will flow from it.

L E M M A I.

Let EAe(TAB.XX.fig.1.)reprefent a cone continued infinitely; let A be the vertex, and Bb and Dd planes parallel to the bafe; and let the cone be filled with uniform matter, whofe particles repel each other with a force inverfely as the n power of the diftance. If n is greater than 3, the force with which a particle

at

at A is repelled by $E B b e$ or all that part of the cone beyond $B b$ is as $\frac{1}{A B^{n-3}}$.

For fuppofing $A B$ to flow, the fluxion of $E B b e$ is proportional to $- \dot{A B} \times A B^2$, and the fluxion of its repulfion on A is proportional to $\frac{-\dot{A B}}{A B^{n-2}}$; the fluent of which is $\frac{1}{n-3 \times A B^{n-3}}$; which when $A B$ is infinite is equal to nothing; confequently the repulfion of $E B b e$ is proportional to $\frac{1}{n-3 \times A B^{n-3}}$ or to $\frac{1}{A B^{n-3}}$.

COROLLARY.

If $A B$ is infinitely fmall, $\frac{1}{A B^{n-3}}$ is infinitely great; therefore the repulfion of that part of the cone between A and $B b$, on A, is infinitely greater than the repulfion of all that beyond it.

LEMMA II.

By the fame method of reafoning it appears, that if n is equal to 3, the repulfion of the matter between $B b$ and $D d$ on a particle at A, is proportional to the logarithm of $\frac{A D}{A B}$; confequently, the repulfion of that part is infinitely fmall in refpect of that between A and $B b$, and alfo infinitely fmall in refpect of that beyond $D d$.

LEMMA

Lemma III.

In like manner, if n is lefs than 3, the repulfion of the part between A and Bb on A is proportional to AB^{3-n}: confequently the repulfion of the matter between A and Bb on A, is infinitely fmall in refpect of that beyond it.

Corollary.

It is eafy to fee from thefe three lemmata, that, if the electric attraction and repulfion had been fuppofed to be inverfely, as fome higher power of the diftance than the cube; a particle could not have been fenfibly affected by the repulfion of any fluid, except what was placed clofe to it. If the repulfion was inverfely, as the cube of the diftance, a particle could not be fenfibly affected by the repulfion of any finite quantity of fluid, except what was clofe to it. But as the repulfion is fuppofed to be inverfely as fome power of the diftance lefs than the cube, a particle may be fenfibly affected by the repulfion of a finite quantity of fluid, placed at any finite diftance from it.

Definition.

If the electric fluid in any body, is by any means confined in fuch manner that it cannot move from one part of the body to the other; I call it immoveable: if it is able to move readily from one part to another, I call it moveable.

PROPOSITION I.

A body overcharged with electric fluid attracts or repels a particle of matter or fluid, and is attracted or repelled by it, with exactly the same force as it would, if the matter in it, together with so much of the fluid as is sufficient to saturate it, was taken away, or as if the body consisted only of the redundant fluid in it. In like manner an undercharged body attracts or repels with the same force, as if it consisted only of the redundant matter; the electric fluid, together with so much of the matter as is sufficient to saturate it, being taken away.

This is evident from the definition of saturation.

PROP. II.

Two over or undercharged bodies attract or repel each other with just the same force that they would, if each body consisted only of the redundant fluid in it, if overcharged, or of the redundant matter in it, if undercharged.

For, let the two bodies be called A and B; by the last proposition the redundant substance in B impels each particle of fluid and matter in A, and consequently impels the whole body A, with the same force that the whole body B impels it: for the same reason the redundant substance in A impels the redundant substance in B, with the same force that
the

the whole body A impels it. It is ſhewn therefore, that the whole body B impels the whole body A, with the ſame force that the redundant ſubſtance in B impels the whole body A, or with which the whole body A impels the redundant ſubſtance in B; and that the whole body A impels the redundant ſubſtance in B, with the ſame force that the redundant ſubſtance in A impels the redundant ſubſtance in B; therefore the whole body B impels the whole body A, with the ſame force with which the redundant ſubſtance in A impels the redundant ſubſtance in B, or with which the redundant ſubſtance in B impels the redundant ſubſtance in A.

COROLLARY.

Let the matter in all the reſt of ſpace, except in two given bodies, be ſaturated with immoveable fluid; and let the fluid in thoſe two bodies be alſo immoveable. Then, if one of the bodies is ſaturated, and the other either over or undercharged, they will not at all attract or repel each other.

If the bodies are both overcharged, they will repel each other.

If they are both undercharged, they will alſo repel each other.

If one is overcharged and the other undercharged, they will attract each other.

N. B. In this corollary, when I call a body over-charged, I would be underſtood to mean, that it is overcharged in all parts, or at leaſt no where under-

undercharged : in like manner, when I call it
undercharged, I mean that it is undercharged in
all parts, or at leaſt no where overcharged.

P R O P. III.

If all the bodies in the univerſe are ſaturated with
electric fluid, it is plain that no part of the fluid
can have any tendency to move.

P R O P. IV.

If the quantity of electric fluid in the univerſe is
exactly ſufficient to ſaturate the matter therein,
but unequally diſperſed, ſo that ſome bodies are
overcharged and others undercharged; then, if
the electric fluid is not confined, it will immedi-
ately move till all the bodies in the univerſe are
ſaturated.

For, ſuppoſing that any body is overcharged, and
the bodies near it are not, a particle at the ſurface of
that body will be repelled from it by the redundant
fluid within ; conſequently ſome fluid will run out
of that body; but if the body is undercharged, a
particle at its ſurface will be attracted towards the
body by the redundant matter within, ſo that ſome
fluid will run into the body.

N. B. In Prob. IV. Caſe III. there will be ſhewn
an exception to this propoſition; there may
perhaps be ſome other exceptions to it: but I
3   think

think there can be no doubt, but what this proposition muft hold good in general.

Lemma IV.

Let B D E, *bde*, and $\beta \delta \epsilon$ (fig. 2.) be concentric fpherical furfaces, whofe center is C : if the fpace * B *b* is filled with uniform matter, whofe particles repel with a force inverfely, as the fquare of the diftance, a particle placed any where within the fpace C *b*, as at P, will be repelled with as much force in one direction as another, or it will not be impelled in any direction. This is demonftrated in Newt. Princip. liber I. prop. lxx. It follows alfo from his demonftration, that if the repulfion is inverfely, as fome higher power of the diftance than the fquare, the particle P will be impelled towards the center ; and if the repulfion is inverfely as fome lower power than the fquare, it will be impelled from the center.

Lemma V.

If the repulfion is inverfely as the fquare of the diftance, a particle placed any where without the fphere B D E, is repelled by that fphere, and alfo by the fpace B *b*, with the fame force that it would if all the matter therein was collected in the center of the

* By the fpace B *b* or B β, I mean the fpace comprehended between the fpherical furfaces B D E and *bde*, or between B D E and $\beta \delta \epsilon$: by the fpace C *b* or C β, I mean the fpheres *bde* or $\beta \delta \epsilon$.

fphere ;

sphere ; provided the denfity of the matter therein is every where the fame at the fame diftance from the center. This is eafily deduced from prop. 71. of the fame book, and has been demonftrated by other authors.

PROP. V.

PROBLEM 1. Let the fphere B D E be filled with uniform folid matter, overcharged with electric fluid : let the fluid therein be moveable, but unable to efcape from it : let the fluid in the reft of infinite fpace be moveable, and fufficient to faturate the matter therein ; and let the matter in the whole of infinite fpace, or at leaft in the fpace B β, whofe dimenfions will be given below, be uniform and folid ; and let the law of the electric attraction and repulfion be inverfely as the fquare of the diftance : it is required to determine in what manner the fluid will be difpofed both within and without the globe.

Take the fpace B b fuch, that the interftices between the particles of matter therein fhall be juft fufficient to hold a quantity of electric fluid, whofe particles are preffed clofe together, fo as to touch each other, equal to the whole redundant fluid in the globe, befides the quantity requifite to faturate the matter in B b; and take the fpace B β fuch, that the matter therein fhall be juft able to faturate the redundant fluid in the globe : then, in all parts of the fpace B b, the fluid will be preffed clofe together, fo

that its particles shall touch each other; the space
Bβ will be intirely deprived of fluid; and in the
space C b, and all the rest of infinite space, the mat-
ter will be exactly saturated.

For, if the fluid is disposed in the above-mentioned
manner, a particle of fluid placed anywhere within
the space C b will not be impelled in any direction
by the fluid in B b, or the matter in B β, and will
therefore have no tendency to move: a particle
placed anywhere without the sphere β δ ε will be at-
tracted with just as much force by the matter in B β,
as it is repelled by the redundant fluid in B b, and
will therefore have no tendency to move: a particle
placed anywhere within the space B b, will indeed
be repelled towards the surface, by all the redundant
fluid in that space which is placed nearer the center
than itself; but as the fluid in that space is already
pressed as close together as possible, it will not have
any tendency to move; and in the space B β there is
no fluid to move, so that no part of the fluid can
have any tendency to move.

Moreover, it seems impossible for the fluid to be
at rest, if it is disposed in any other form; for
if the density of the fluid is not everywhere the same
at the same distance from the center, but is greater
near b than near d, a particle placed anywhere between
those two points will move from b towards d; but
if the density is everywhere the same at the same
distance from the center, and the fluid in B b is not
pressed close together, the space C b will be over-
charged, and consequently a particle at b will be re-
pelled from the center, and cannot be at rest: in
like manner, if there is any fluid in B β, it cannot

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be at reſt: and, by the ſame kind of reaſoning, it might be ſhewn, that, if the fluid is not ſpread uniformly within the ſpace C *b*, and without the ſphere β δ ε, it cannot be at reſt.

COROLLARY I.

If the globe B D E is undercharged, every thing elſe being the ſame as before, there will be a ſpace B *b*, in which the matter will be intirely deprived of fluid, and a ſpace B β, in which the fluid will be preſſed cloſe together; the matter in B *b* being equal to the whole redundant matter in the globe, and the redundant fluid in B β, being juſt ſufficient to ſaturate the matter in B *b*: and in all the reſt of ſpace the matter will be exactly ſaturated. The demonſtration is exactly ſimilar to the foregoing.

COROL. II.

The fluid in the globe B D E will be diſpoſed in exactly the ſame manner, whether the fluid without is immoveable, and diſpoſed in ſuch manner, that the matter ſhall be everywhere ſaturated, or whether it is diſpoſed as above deſcribed; and the fluid without the globe will be diſpoſed in juſt the ſame manner, whether the fluid within is diſpoſed uniformly, or whether it is diſpoſed as above deſcribed.

PROP.

P R O P. VI.

Prob. 2. To determine in what manner the fluid will be difpofed in the globe BDE, fuppofing every thing as in the laft problem, except that the fluid on the outfide of the globe is immoveable, and difpofed in fuch manner as everywhere to faturate the matter, and that the electric attraction and repulfion is inverfely, as fome other power of the diftance than the fquare.

I am not able to anfwer this problem accurately; but I think we may be certain of the following circumftances.

Case 1. Let the repulfion be inverfely as fome power of the diftance between the fquare and the cube, and let the globe be overcharged.
It is certain that the denfity of the fluid will be everywhere the fame, at the fame diftance from the center. Therefore, firft, There can be no fpace as C *b*, within which the matter will be everywhere faturated; for a particle at *b* is impelled towards the center, by the redundant fluid in B *b*, and will therefore move towards the center, unlefs C *b* is fufficiently overcharged to prevent it. Secondly, The fluid clofe to the furface of the fphere will be preffed clofe together; for otherwife a particle fo near to it, that the quantity of fluid between it and the furface fhould be very fmall, would move towards it; as the repulfion of the fmall quantity of fluid between

it

it and the furface, would be unable to balance the repulfion of the fluid on the other fide. Whence, I think, we may conclude, that the denfity of the fluid will increafe gradually from the center to the furface, where the particles will be preffed clofe together: whether the matter exactly at the center will be overcharged, or only faturated, I cannot tell.

COROLLARY.

For the fame reafon, if the globe be undercharged, I think we may conclude, that the denfity of the fluid will diminifh gradually from the center to the furface, where the matter will be entirely deprived of fluid.

CASE 2. Let the repulfion be inverfely as fome power of the diftance lefs than the fquare; and let the globe be overcharged.

There will be a fpace B *b*, in which the particles of the fluid will be everywhere preffed clofe together; and the quantity of redundant fluid in that fpace will be greater than the quantity of redundant fluid in the whole globe B D E; fo that the fpace C *b*, taken all together, will be undercharged: but I cannot tell in what manner the fluid will be difpofed in that fpace.

For it is certain, that the denfity of the fluid will be everywhere the fame, at the fame diftance from the center. Therefore, let *b* be any point where the fluid is not preffed clofe together, then will a particle at *b* be impelled towards the furface, by the

redundant

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redundant fluid in the fpace B*b*; therefore, unlefs the fpace C*b* is undercharged, the particle will move towards the furface.

COROLLARY.

For the fame reafon, if the globe is undercharged, there will be a fpace B*b*, in which the matter will be intirely deprived of fluid, the quantity of matter therein being more than the whole redundant matter in the globe; and, confequently, the fpace C*b*, taken all together, will be overcharged.

LEMMA VI.

Let the whole fpace comprehended between two parallel planes, infinitely extended each way, be filled with uniform matter, the repulfion of whofe particles is inverfely as the fquare of the diftance; the plate of matter formed thereby will repel a particle of matter with exactly the fame force, at whatever diftance from it, it be placed.

For, fuppofe that there are two fuch plates, of equal thicknefs, placed parallel to each other, let A (fig. 3.) be any point not placed in or between the two plates: let B C D reprefent any part of the neareft plate: draw the lines A B, A C, and A D, cutting the furtheft plate in *b*, *c*, and *d*; for it is plain, that if they cut one plate, they muft, if produced, cut the other : the triangle B C D is to the triangle *b c d*, as A B² to A *b*²; therefore a particle of matter at A will be repelled with the fame force by the matter in the triangle B C D, as by that in *b c d*. Whence it appears, that a particle at A will
be

be repelled with as much force by the neareſt plate, as by the more diſtant; and conſequently, will be impelled with the ſame force by either plate, at whatever diſtance from it it be placed.

COROLLARY.

If the repulſion of the particles is inverſely as ſome higher power of the diſtance than the ſquare, the plate will repel a particle with more force, if its diſtance be ſmall than if it be great; and if the repulſion is inverſely as ſome lower power than the ſquare, it will repel a particle with leſs force, if its diſtance be ſmall than if it be great.

PROP. VII.

PROB. 3. In fig. 4. let the parallel lines A a, B b, &c. repreſent parallel planes infinitely extended each way: let the ſpaces * A D and E H be filled with uniform ſolid matter: let the electric fluid in each of thoſe ſpaces be moveable and unable to eſcape : and let all the reſt of the matter in the univerſe be ſaturated with immoveable fluid ; and let the electric attraction and repulſion be inverſely as the ſquare of the diſtance. It is required to determine in what manner the fluid will be diſpoſed in the ſpaces A D and E H, according as one or both of them are over or undercharged.

* By the ſpace AD or AB, &c. I mean the ſpace comprehended between the planes A a and D d, or between A a and B b.

Let

Let A D be that fpace which contains the greateſt quantity of redundant fluid, if both fpaces are overcharged, or which contains the leaſt redundant matter, if both are undercharged; or, if one is overcharged, and the other undercharged, let A D be the overcharged one. Then, firſt, There will be two fpaces, A B and G H, which will either be intirely deprived of fluid, or in which the particles will be preſſed clofe together; namely, if the whole quantity of fluid in A D and E H together, is leſs than fuffic:ent to faturate the matter therein, they will be intirely deprived of fluid; the quantity of redundant matter in each being half the whole redundant matter in A D and E H together: but if the fluid in A D and E H together is more than fufficient to faturate the matter, the fluid in A B and G H will be preſſed clofe together; the quantity of redundant fluid in each being half the whole redundant fluid in both fpaces. 2dly, In the fpace C D the fluid will be preſſed clofe together; the quantity of fluid therein being fuch, as to leave juſt enough fluid in B C to faturate the matter therein. 3dly, The fpace E F will be intirely deprived of fluid; the quantity of matter therein being fuch, that the fluid in F G ſhall be juſt fufficient to faturate the matter therein: confequently, the redundant fluid in C D will be juſt fufficient to faturate the redundant matter in E F; for as A B and G H together contain the whole redundant fluid or matter in both fpaces, the fpaces B D and E G together contain their natural quantity of fluid; and therefore, as B C and F G each contain their natural quantity of fluid, the fpaces C D and E F together contain their

natural

natural quantity of fluid. And, 4thly, The spaces
B C and F G will be saturated in all parts.

For, first, If the fluid is disposed in this manner,
no particle of it can have any tendency to move:
for a particle placed anywhere in the spaces B C and
F G, is attracted with just as much force by E F, as
it is repelled by C D; and it is repelled or attracted
with just as much force by A B, as it is in a con-
trary direction by G H, and, consequently, has no
tendency to move. A particle placed anywhere in
the space C D, or in the spaces A B and G H, if
they are overcharged, is indeed repelled with more
force towards the planes D *d*, A *a*, and H *b*, than it
is in the contrary direction; but as the fluid in those
spaces is already as much compressed as possible, the
particle will have no tendency to move.

2dly, It seems impossible that the fluid should be
at rest, if it is disposed in any other manner: but as
this part of the demonstration is exactly similar to
the latter part of that of Problem the first, I shall
omit it.

Corol. I.

If the two spaces A D and E H are both over-
charged, the redundant fluid in C D is half the dif-
ference of the redundant fluid in those spaces: for
half the difference of the redundant fluid in those
spaces, added to the quantity in A B, which is half
the sum, is equal to the whole quantity in A D. For
a like reason, if A D and E H are both undercharged,
the redundant matter in E F is half the difference of
the redundant matter in those spaces; and if A D is

overcharged, and E H undercharged, the redundant fluid in C D exceeds half the redundant fluid in A D, by a quantity fufficient to faturate half the redundant matter in E H.

COROL. II.

It was before faid, that the fluid in the fpaces A B and G H (when there is any fluid in them) is repelled againft the planes A *a* and H *b*; and, confequently, would run out through thofe planes, if there was any opening for it to do fo. The force with which the fluid preffes againft the planes A *a* and H *b*, is that with which the redundant fluid in A B is repelled by that in G H; that is, with which half the redundant fluid in both fpaces is repelled by an equal quantity of fluid. Therefore, the preffure againft A *a* and H *b* depends only on the quantity of redundant fluid in both fpaces together, and not at all on the thicknefs or diftance of thofe fpaces, or on the proportion in which the fluid is divided between the two fpaces. If there is no fluid in A B and G H, a particle placed on the outfide of the fpaces A D and E H, contiguous to the planes A *a* or H *b*, is attracted towards thofe planes by all the matter in A B and G H, *id eft*, by all the redundant matter in both fpaces; and, confequently, endeavours to infinuate itfelf into the fpace A D or E H; and the force with which it does fo, depends only on the quantity of redundant matter in both fpaces together. The fluid in C D alfo preffes againft the plane D *d*, and the force with which it does fo, is that with which

which the redundant fluid in C D is attracted by the matter in E F.

COROL. III.

If A D is overcharged, and E H undercharged, and the redundant fluid in A D is exactly sufficient to saturate the redundant matter in E H, all the redundant fluid in A D will be collected in the space C D, where it will be pressed close together : the space E F will be intirely deprived of fluid, the quantity of matter therein being just sufficient to saturate the redundant fluid in C D, and the spaces A C and F H will be everywhere saturated. Moreover, if an opening is made in the planes A *a* or H *b*, the fluid within the spaces A D or E H will have no tendency to run out thereat, nor will the fluid on the outside have any tendency to run in at it : a particle of fluid too placed anywhere on the outside of both spaces, as at P, will not be at all attracted or repelled by those spaces, any more than if they were both saturated ; but a particle placed anywhere between those spaces, as at S, will be repelled from *d* towards *e* ; and if a communication was made between the two spaces, by the canal *d e*, the fluid would run out of A D into E H, till they were both saturated.

PROP.

P R O P. VIII.

PROB. 4. To determine in what manner the fluid will be difpofed in the fpace A D, fuppofing that all the reft of the univerfe is faturated with immoveable fluid, and that the electric attraction and repulfion is inverfely as fome other power of the diftance than the fquare.

I am not able to anfwer this Problem accurately, except when the repulfion is inverfely as the fimple or fome lower power of the diftance; but I think we may be certain of the following circumftances.

CASE 1. Let the repulfion be inverfely as fome power of the diftance between the fquare and the cube, and let A D be overcharged.

Firft, It is certain that the denfity of the fluid muft be everywhere the fame, at the fame diftance from the planes A*a* and D*d*. 2dly, There can be no fpace as BC, of any fenfible breadth, in which the matter will not be overcharged. And, 3dly, The fluid clofe to the planes A *a* and D *d* will be preffed clofe together. Whence, I think, we may conclude, that the denfity of the fluid will increafe gradually from the middle of the fpace to the outfide, where it will be preffed clofe together. Whether the matter exactly in the middle will be overcharged, or only faturated, I cannot tell.

CASE 2. Let the repulfion be inverfely as fome power of the diftance between the fquare and the fimple power, and let A D be overcharged.

There will be two fpaces A B and D C, in which the fluid will be preffed clofe together, and the quantity of redundant fluid in each of thofe fpaces will be more than half the redundant fluid in A D ; fo that the fpace B C, taken all together, will be undercharged ; but I cannot tell in what manner the fluid will be difpofed in that fpace. The demonftrations of thefe two cafes are exactly fimilar to thofe of the two cafes of Prob. 2.

CASE 3. If the repulfion is inverfely as the fimple or fome lower power of the diftance, and A D is overcharged, all the fluid will be collected in the fpaces A B and C D, and B C will be intirely deprived of fluid. If A D contains juft fluid enough to faturate it, and the repulfion is inverfely as the diftance, the fluid will remain in equilibrio, in whatever manner it is difpofed ; provided its denfity is everywhere the fame, at the fame diftance from the planes A a and D d: but if the repulfion is inverfely as fome lefs power than the fimple one, the fluid will be in equilibrio, whether it is either fpread uniformly, or whether it is all collected in that plane which is in the middle between A a and D d, or whether it is all collected in the fpaces A B and C D ; but not, I believe, if it is difpofed in any other manner.

The demonftration depends upon this circumftance ; namely, that if the repulfion is inverfely as the diftance, two fpaces A B and C D, repel a particle

ticle, placed either between them, or on the outfide of them, with the fame force as if all the matter of thofe fpaces was collected in the middle plane between them.

It is needlefs mentioning the three cafes in which A D is undercharged, as the reader will eafily fupply the place.

Though the four foregoing problems do not immediately tend to explain the phænomena of electricity, I chofe to infert them ; partly becaufe they feem worth engaging our attention in themfelves ; and partly becaufe they ferve, in fome meafure, to confirm the truth of fome of the following propofitions, in which I am obliged to make ufe of a lefs accurate kind of reafoning.

In the following propofitions, I fhall always fuppofe the bodies I fpeak of to confift of folid matter, confined to the fame fpot, fo as not to be able to alter its fhape or fituation by the attraction or repulfion of other bodies on it : I fhall alfo fuppofe the electric fluid in thefe bodies to be moveable, but unable to efcape, unlefs when otherwife expreffed. As for the matter in all the reft of the univerfe, I fhall fuppofe it to be faturated with immoveable fluid. I fhall alfo fuppofe the electric attraction and repulfion to be inverfely as any power of the diftance lefs than the cube, except when otherwife expreffed.

By a canal, I mean a flender thread of matter, of fuch kind that the electric fluid fhall be able to move readily along it, but fhall not be able to efcape from it, except at the ends, where it communicates with other bodies. Thus, when I fay that two bodies

com-

communicate with each other by a canal, I mean that the fluid fhall be able to pafs readily from one body to the other by that canal.

P R O P. IX.

> If any body at a diftance from any over or under-charged body be overcharged, the fluid within it will be lodged in greater quantity near the furface of the body than near the. center. For, if you fuppofe it to be fpread uniformly all over the body, a particle of fluid in it, near the furface, will be repelled towards the furface, by a greater quantity of fluid than that by which it is re-pelled from it; confequently, the fluid will flow towards the furface, and make it denfer there : moreover, the particles of fluid clofe to the furface will be preffed clofe together; for otherwife, a particle placed fo near it, that the quantity of redundant fluid between it and the furface fhould be very fmall, would move to-wards it; as the fmall quantity of redundant fluid between it and the furface would be un-able to balance the repulfion of that on the other fide.

From the four foregoing problems it feems likely, that if the electric attraction or repulfion is inverfely as the fquare of the diftance, almoft all the redun-dant fluid in the body will be lodged clofe to the furface, and there preffed clofe together, and the reft of the body will be faturated. If the repulfion is inverfely as fome power of the diftance between

the

the fquare and the cube, it is likely that all parts of the body will be overcharged : and if it is inverfely as fome lefs power than the fquare, it is likely that all parts of the body, except thofe near the furface, will be undercharged.

COROLLARY.

For the fame reafon, if the body is undercharged, the deficiency of fluid will be greater near the furface than near the center, and the matter near the furface will be intirely deprived of fluid. It is likely too, if the repulfion is inverfely as fome higher power of the diftance than the fquare, that all parts of the body will be undercharged : if it is inverfely as the fquare, that all parts, except near the furface, will be faturated : and if it is inverfely as fome lefs power than the fquare, that all parts, except near the fur-face, will be overcharged.

PROP. X.

Let the bodies A and D (fig. 5.) communicate with each other, by the canal E F ; and let one of them, as D, be overcharged ; the other body A will be fo alfo.

For as the fluid in the canal is repelled by the re-dundant fluid in D, it is plain, that unlefs A was overcharged, fo as to balance that repulfion, the fluid would run out of D into A.

In like manner, if one is undercharged, the òther muft be fo too.

PROP.

P R O P. XI.

Let the body A (fig. 6.) be either faturated or over or undercharged ; and let the fluid within it be in equilibrio. Let now the body B, placed near it, be rendered overcharged, the fluid within it being fuppofed immoveable, and dif-pofed in fuch manner, that no part of it fhall be undercharged ; the fluid in A will no longer be in equilibrio, but will be repelled from B : therefore, the fluid will flow from thofe parts of A which are neareft to B, to thofe which are more diftant from it ; and, confequently, the part adjacent to M N (that part of the fur-face of A which is turned towards B) will be made to contain lefs electric fluid than it did before, and that adjacent to the oppofite furface R S will contain more than before.

It muft be obferved, that when a fufficient quan-tity of fluid has flowed from M N towards R S, the repulfion which the fluid in the part adjacent to M N exerts on the reft of the fluid in A, will be fo much weakened, and the repulfion of that in the part near R S will be fo much increafed, as to com-penfate the repulfion of B, which will prevent any more fluid flowing from M N to R S.

The reafon why I fuppofe the fluid in B to be immoveable is, that otherwife a queftion might arife, whether the attraction or repulfion of the body A might not caufe fuch an alteration in the difpofition of the fluid in B, as to caufe fome parts of it to be

undercharged; which might make it doubtful, whe-
ther B did on the whole repel the fluid in A.   It is
evident, however, that this propofition would hold
good, though fome parts of B were undercharged,
provided it did on the whole repel the fluid in A.

COROLLARY.

If B had been made undercharged, inftead of over-
charged, it is plain that fome fluid would have
flowed from the further part R S to the nearer part
M N, inftead of from M N to R S.

PROP. XII.

Let us now fuppofe that the body A commu-
nicates by the canal E F, with another body
D, placed on the contrary fide of it from B, as
in fig. 5 ; and let thefe two bodies be either fa-
turated, or over or undercharged ; and let the
fluid within them be in equilibrio.   Let now
the body B be overcharged : it is plain that
fome fluid will be driven from the nearer part
M N to the further part R S, as in the former
propofition ; and alfo fome fluid will be driven
from R S, through the canal, to the body D ;
fo that the quantity of fluid in D will be in-
creafed thereby, and the quantity in A, taking
the whole body together, will be diminifhed ;
the quantity in the part near M N will alfo be
diminifhed ; but whether the quantity in the
part near R S will be diminifhed or not, does
not appear for certain ; but I fhould imagine it
would be not much altered.

COROL-

COROLLARY.

In like manner, if B is made undercharged, some fluid will flow from D to A, and also from that part of A near R S, to the part near M N.

PROP. XIII.

Suppose now that the bodies A and D communicate by the bent canal M P N *n p m* (fig. 7.) instead of the straight one E F : let the bodies be either saturated or over or undercharged as before; and let the fluid be at rest; then if the body B is made overcharged, some fluid will still run out of A into D; provided the repulsion of B on the fluid in the canal is not too great.

The repulsion of B on the fluid in the canal, will at first drive some fluid out of the leg M P *p m* into A, and out of N P *p n* into D, till the quantity of fluid in that part of the canal which is nearest to B is so much diminished, and its repulsion on the rest of the fluid in the canal is so much diminished also as to compensate the repulsion of B : but as the leg N P *p n* is longer than the other, the repulsion of B on the fluid in it will be greater; consequently some fluid will run out of A into D, on the same principle that water is drawn out of a vessel through a syphon : but if the repulsion of B on the fluid in the canal is so great, as to drive all the fluid out of the space G P H *p* G, so that the fluid in the leg M G *p m* does not

join

join to that in N H *p n* ; then it is plain that no fluid can run out of A into D ; any more than water will run out of a veffel through a fyphon, if the height of the bend of the fyphon above the water in the veffel, is greater than that to which water will rife in vacuo.

COROLLARY.

If B is made undercharged, fome fluid will run out of D into A ; and that though the attraction of B on the fluid in the canal is ever fo great.

PROP. XIV.

Let ABC (fig. 8.) be a body overcharged with immoveable fluid, uniformly fpread ; let the bodies near ABC on the outfide be faturated with immoveable fluid ; and let D be a body inclofed within ABC, and communcating by the canal DG with other diftant bodies faturated with fluid ; and let the fluid in D and the canal and thofe bodies be moveable ; then will the body D be rendered undercharged.

For let us firft fuppofe that D and the canal are faturated, and that D is nearer to B than to the oppofite part of the body, C ; then will all the fluid in the canal be repelled from C by the redundant fluid in ABC ; but if D is nearer to C than to B, take the point F, fuch that a particle placed there would be repelled from C with as much force as one at D is repelled towards C ; the fluid in DF, taking the whole

whole together, will be repelled with as much force one way as the other; and the fluid in FG is all of it repelled from C : therefore in both cases the fluid in the canal, taking the whole together, is repelled from C; consequently some fluid will run out of D and the canal, till the attraction of the unsaturated matter therein is sufficient to balance the repulsion of the redundant fluid in ABC.

PROP. XV.

If we now suppose that the fluid on the outside of ABC is moveable; the matter adjacent to ABC on the outside, will become undercharged. I see no reason however to think that that will prevent the body D from being undercharged; but I cannot say exactly what effect it will have, except when ABC is spherical and the repulsion is inversely as the square of the distance; in this case it appears by Prob. I. that the fluid in the part DB of the canal will be repelled from C, with just as much force as in the last proposition; but the fluid in the part BG will not be repelled at all: consequently D will be undercharged, but not so much as in the last proposition.

COROLLARY.

If ABC is now supposed to be undercharged, it is certain that D will be overcharged, provided the matter near ABC on the outside is saturated with im-

moveable

moveable fluid; and there is great reafon to think
that it will be fo, though the fluid in that matter is
moveable.

P R O P. XVI.

Let A E F B (fig. 9.) be a long cylindric
body, and D an undercharged body; and let
the quantity of fluid in A E F B be fuch, that
the part near E F fhall be faturated. It appears
from what has been faid before, that the part
near A B will be overcharged; and moreover
there will be a certain fpace, as A *a b* B, adjoin-
ing to the plane A B, in which the fluid will be
preffed clofe together; and the fluid in that
fpace will prefs againft the plane A B, and will
endeavour to efcape from it; and by Prop. II.
the two bodies will attract each other: now I
fay that the force with which the fluid preffes
againft the plane A B, is very nearly the fame
with which the two bodies attract each other in
the direction E A; provided that no part of
A E F B is undercharged.

Suppofe fo much of the fluid in each part of the
cylinder as is fufficient to faturate the matter in that
part, to become folid; the remainder, or the redun-
dant fluid remaining fluid as before. In this cafe the
preffure againft the plane A B muft be exactly equal
to that with which the two bodies attract each other,
in the direction E A: for the force with which D
attracts that part of the fluid which we fuppofed to
become folid, is exactly equal to that, with which it
repels

repels the matter in the cylinder; and the redundant
fluid in E *a b* F is at liberty to move, if it had any
tendency to do fo, without moving the cylinder; fo
that the only thing which has any tendency to impel
the cylinder in the direction E A is the preffure of the
redundant fluid in A *a b* B againft A B; and as the
part near E F is faturated, there is no redundant fluid
to prefs againft the plane E F, and thereby to coun-
teract the preffure againft A B. Suppofe now all the
electric fluid in the cylinder to become fluid; the
force with which the two bodies attract each other
will remain exactly the fame; and the only alteration
in the preffure againft A B, will be, that that part of
the fluid in A *a b* B, which we at firft fuppofed folid
and unable to prefs againft the plane, will now be at
liberty to prefs againft it; but as the denfity of the
fluid when its particles are preffed clofe together
may be fuppofed many times greater than when it is
no denfer than fufficient to faturate the matter in the
cylinder, and confequently the quantity of redun-
dant fluid in A *a b* B many times greater than that
which is required to faturate the matter therein, it
follows that the preffure againft A B will be very
little more than on the firft fuppofition.

N. B. If any part of the cylinder is undercharged,
the preffure againft A B is greater than the force
with which the bodies attract. If the electric
repulfion is inverfely as the fquare or fome
higher power of the diftance, it feems very
unlikely that any part of the cylinder fhould
be undercharged; but if the repulfion is in-
verfely as fome lower power than the fquare, it

is

is not improbable but fome part of the cylinder
may be undercharged.

Lemma VII.

Let AB (fig. 10.) reprefent an infinitely thin
flat circular plate, feen edgeways, fo as to appear
to the eye as a ftraight line; let C be the center of
the circle; and let DC paffing through C, be per-
pendicular to the plane of the plate; and let the
plate be of uniform thicknefs, and confift of uniform
matter, whofe particles repel with a force inverfely as
the n power of the diftance; n being greater than
one, and lefs than three: the repulfion of the plate
on a particle at D is proportional to $\dfrac{DC}{DC^{n-1}} - \dfrac{DC}{DA^{n-1}}$;
provided the thicknefs of the plate and fize of the
particle D is given.

For if CA is fuppofed to flow, the correfponding
fluxion of the quantity of matter in the plate, is pro-
portional to $CA \times C\dot{A}$; and the correfponding fluxion
of the repulfion of the plate on the particle D, in the
direction DC, is proportional to $\dfrac{CA \times C\dot{A}}{DA^n} \times \dfrac{DC}{DA}$, $=$
$\dfrac{D\dot{A} \times DC}{DA^n}$; for $D\dot{A}$ is to $C\dot{A} :: CA : DA$; the va-
riable part of the fluent of which is $\dfrac{-DC}{n-1 \times DA^{n-1}}$:
whence the repulfion of the plate on the particle D
is proportional to $\dfrac{DC}{n-1 \times DC^{n-1}} - \dfrac{DC}{n-1 \times DA^{n-1}}$, or to
$\dfrac{DC}{DC^{n-1}} - \dfrac{DC}{DA^{n-1}}$.

COROLLARY.

If DC^{n-1} is very fmall in refpect of CA^{n-1}, the particle D is repelled with very nearly the fame force as if the diameter of the plate was infinite.

LEMMA VIII.

Let L and l reprefent the two legs of a right angled triangle, and b the hypothenufe; if the fhorter leg l is fo much lefs than the other, that l^{n-1} is very fmall in refpect of L^{n-1}, $b^{3-n} - L^{3-n}$ will be very fmall in refpect of l^{3-n}.

For $b^{3-n} = \overline{L^2 + l^2}\Big]^{\frac{3-n}{2}}$, $= L^{3-n} \times \overline{1 + \frac{l^2}{L^2}}\Big]^{\frac{3-n}{2}}$, $=$

$L^{3-n} \times 1 + \dfrac{3-n \times l^2}{2L^2} - \dfrac{3-n \times n-1 \times l^4}{8L^4}$, &c. therefore

$b^{3-n} - L^{3-n} = \dfrac{3-n \times l^2}{2L^{n-1}} - \dfrac{3-n \times n-1 \times l^4}{8L^{n+1}}$, &c. $=$

$\dfrac{l^{3-n} \times 3-n \times l^{n-1}}{2L^{n-1}} - \dfrac{l^{3-n} \times 3-n \times n-1 \times l^{n+1}}{8L^{n+1}}$, &c. which

is very fmall in refpect of l^{3-n}; as l^{n-1} is by the fuppofition very fmall in refpect of L^{n-1}.

LEMMA IX.

Let D C now reprefent the axis of a cylindric or prifmatic column of uniform matter; and let the diameter of the column be fo fmall, that the re-pulfion of the plate A B on it fhall not be fenfibly different from what it would be, if all the matter

in it was collected in the axis: the force with which
the plate repells the column, is proportional to
$DC^{3-n}+AC^{3-n}-DA^{3-n}$; suppofing the thickneſs
of the plate and baſe of the column to be given.

For, if DC is ſuppoſed to flow, the correſponding
fluxion of the repulſion is proportional to $\dfrac{D\dot{C}}{DC^{n-2}}$ —

$\dfrac{DC \times D\dot{C}}{DA^{n-1}} = \dfrac{D\dot{C}}{DC^{n-2}} - \dfrac{D\dot{A}}{DA^{n-2}}$; the fluent of which,

$\dfrac{AC^{3-n}+DC^{3-n}-DA^{3-n}}{3-n}$, vaniſhes when DC va-
niſhes.

COROLL. I.

If the length of the column is ſo great that AC^{n-1}
is very ſmall in reſpect of DC^{n-1}, the repulſion of
the plate on it is very nearly the ſame as if the
column was infinitely continued.

For by Lemma 8, $AC^{3-n}+DC^{3-n}-DA^{3-n}$
differs very little in this caſe from AC^{3-n}; and if
DC is infinite, it is exactly equal to it.

COROLL. II

If AC^{n-1} is very ſmall in reſpect of DC^{n-1}; and
the point E be taken in DC ſuch that EC^{n-1}, ſhall
be very ſmall in reſpect of AC^{n-1}, the repulſion of
the plate on the ſmall part of the column EC, is, to
its repulſion on the whole column DC, very nearly
as EC^{3-n} to AC^{3-n}.

Lᴇᴍᴍᴀ X.

If we now ſuppoſe all the matter of the plate to be collected in the circumference of the circle, ſo as to form an infinitely ſlender uniform ring, its repulſion on the column DC will be leſs than when the matter is ſpread uniformly all over the plate, in the ratio of

$$\frac{3-n \times AC^2}{2} \times \overline{\frac{1}{AC^{n-1}} - \frac{1}{DA^{n-1}}} \text{ to } DC^{3-n} + AC^{3-n} - DA^{3-n}.$$

For it was before ſaid, that if the matter of the plate be ſpread uniformly, its repulſion on the column will be proportional to $DC^{3-n} + AC^{3-n} - DA^{3-n}$, or may be expreſſed thereby ; let now A C, the ſemi-diameter of the plate, be increaſed by the infinitely ſmall quantity A ċ ; the quantity of matter in the plate will be increaſed by a quantity, which is to the whole, as 2 A ċ to A C ; and the repulſion of the plate on the column, will be increaſed by $3-n \times$

$$A ċ \times AC^{2-n} - A ċ \times \frac{AC}{DA} \times 3-n \times DA^{2-n}, = 3-n$$

$$\times A ċ \times AC \times \overline{\frac{1}{AC^{n-1}} - \frac{1}{DA^{n-1}}} : \text{ therefore if a quan-}$$

tity of matter, which is to the whole quantity in the plate, as 2 A ċ to A C be collected in the circumference, its repulſion on the column DC, will be to that of the whole plate, as $3-n \times A ċ \times AC \times$

$$\overline{\frac{1}{AC^{n-1}} - \frac{1}{DA^{n-1}}}, \text{ to } DC^{3-n} + AC^{3-n} - DA^{3-n}; \text{ and}$$

conſequently the repulſion of the plate when all the matter is collected in its circumference, is to its re-

　　　　　　　　　　pulſion

pulſion when the matter is ſpread uniformly, as $\frac{3-n \times AC^2}{2} \times \overline{\frac{1}{AC^{n-1}} - \frac{1}{DA^{n-1}}}$, to $DC^{3-n} + AC^{3-n}$ $- DA^{3-n}$.

COROLL. I.

If the length of the column is ſo great, that AC^{n-1} is very ſmall in reſpect of DC^{n-1}, the repulſion of the plate, when all the matter is collected in the circumference, is to its repulſion when the matter is ſpread uniformly, very nearly as $\frac{3-n \times AC^{3-n}}{2}$. to AC^{3-n}, or as $3-n$ to 2.

COROLL. II.

If EC^{n-1} is very ſmall in reſpect of AC^{n-1}, the repulſion of the plate on the ſhort column EC, when all the matter in the plate is collected in its circumference, is to its repulſion when the matter is ſpread uniformly, very nearly as $\frac{3-n \times n-1 \times EC^2}{4AC^{n-1}}$ to EC^{3-n}, or as $3-n \times n-1 \times EC^{n-1}$ to $4AC^{n-1}$; and is therefore very ſmall in compariſon of what it is when the matter is ſpread uniformly.

For by the ſame kind of proceſs as was uſed in Lemma 8, it appears, that if EC^2 is very ſmall in reſpect of AC^2, $AC^2 \times \overline{\frac{1}{AC^{n-1}} - \frac{1}{EA^{n-1}}}$ differs very

little.

little from $\dfrac{n-1 \times EC^2}{2EA^{n-1}}$, or from $\dfrac{n-1 \times EC^2}{2AC^{n-1}}$; and if EC^{n-1} is very fmall in refpect of AC^{n-1}, EC^2 is *a fortiori* very fmall in refpect of AC^2.

Coroll. III.

Suppofe now that the matter of the plate is denfer near the circumference than near the middle, and that the denfity at and near the middle is to the mean denfity, or the denfity which it would everywhere be of if the matter was fpread uniformly, as δ to one; the repulfion of the plate on EC will be lefs than if the matter was fpread uniformly, in a ratio approaching much nearer to that of δ to one, than to that of equality.

Coroll. IV.

Let every thing be as in the laft corollary, and let π be taken to one, as the force with which the plate actually repels the column DC (DC^{n-1} being very great in refpect of AC^{n-1}) is to the force with which it would repel it, if the matter was fpread uniformly; the repulfion of the plate on EC will be to its repulfion on DC, in a ratio between that of $EC^{3-n} \times \delta$ to $AC^{3-n} \times \pi$, and that of EC^{3-n} to $AC^{3-n} \times \pi$, but will approach much nearer to the former ratio than to the latter.

LEMMA

L e m m a XI.

In the line DC produced, take C F equal to C A: if all the matter of the plate A B is collected in the circumference, its repulsion on the column CD, infinitely continued, is equal to the repulsion of the same quantity of matter collected in the point F, on the same column.

For the repulsion of the plate on the column in the direction C D, is the same, whether the matter of it be collected in the whole circumference, or in the point A. Suppose it therefore to be collected in A; and let an equal quantity of matter be collected in F; take F G constantly equal to A D; and let A D and F G flow: the fluxion of C D is to the fluxion of F G, as A D to C D; and the repulsion of A on the point D, in the direction C D, is to the repulsion of F on G, as CD to A D; and therefore the fluxion of the repulsion of A on the column C D, in the direction C D, is equal to the fluxion of the repulsion of F on C G; and when A D equals A C, the repulsion of both A and F on their respective columns vanishes; and therefore the repulsion of A on the whole column C D equals that of F on C G; and when C D and C G are both infinitely extended, they may be looked upon as the same column.

P R O P. XVII.

Let two similar bodies, of different sizes, and consisting of different sorts of matter, be both overcharged,

Fig.11.

Fig.12.

Fig.13.

Fig.14.

Fig.15

Fig.16

Fig. 12.

B
F
D G H E

β
b
f
δ d g h e
ε

Fig. 17.

R
A
N
H G C
r
c
B

Fig. 15

E
b β δ d F
B D
P
A

Fig. 18.

D

Fig. 16.

E
P D
A

x
y
z
v
h g

B F

M L

B C D
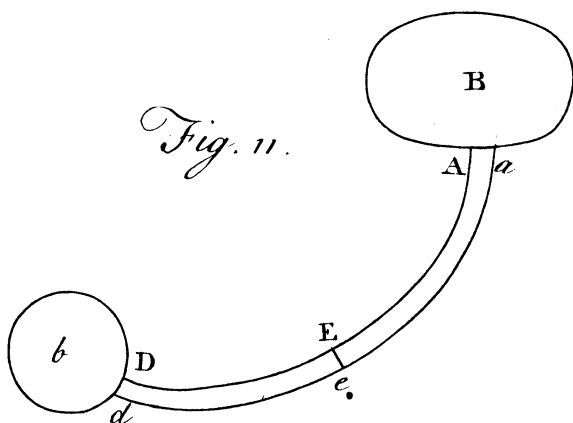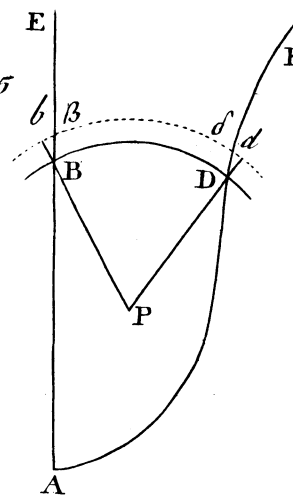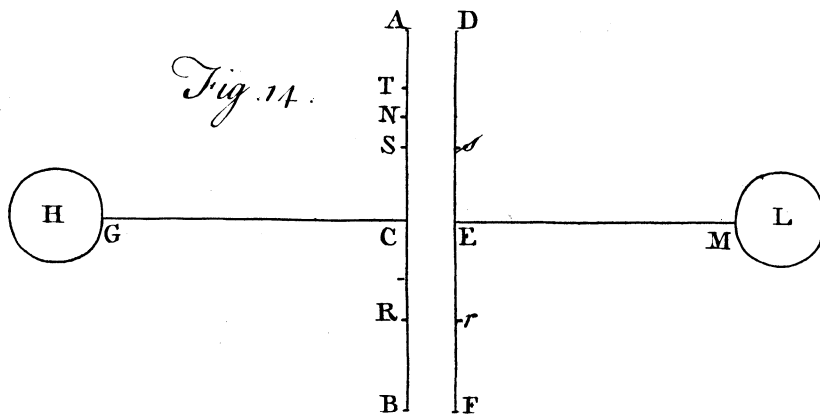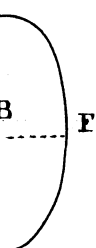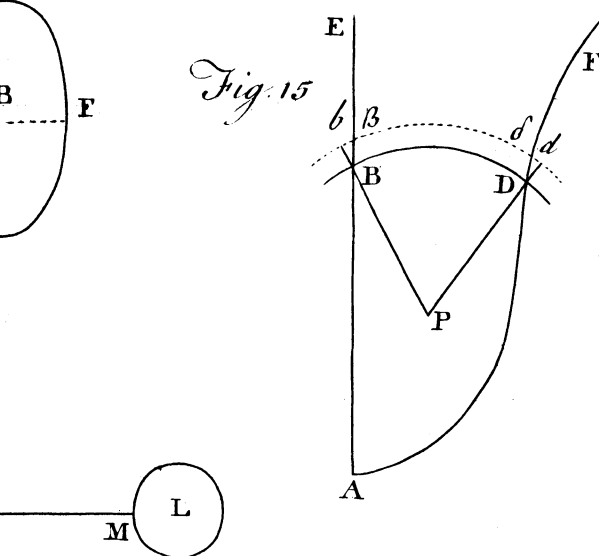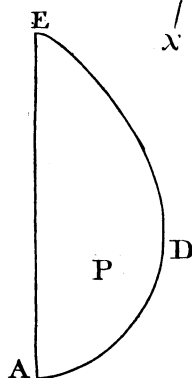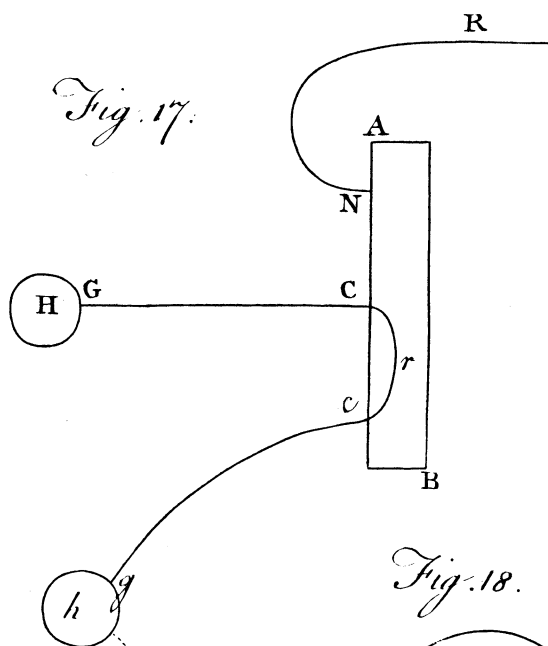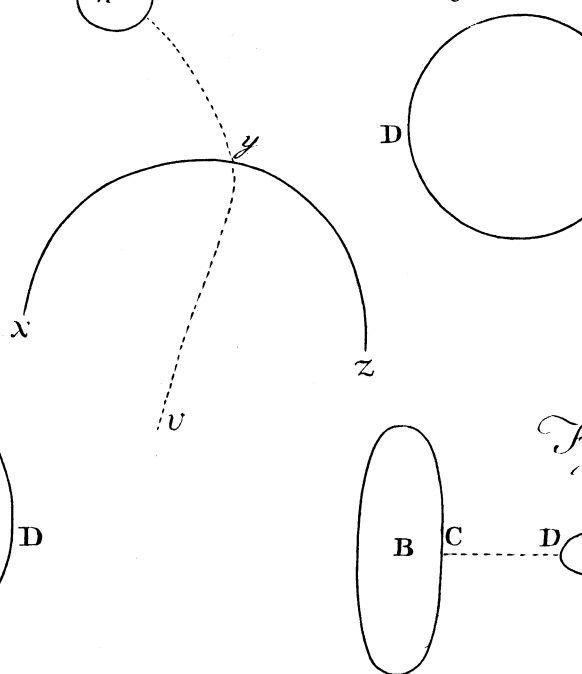
Fig. 17.

Fig. 18.

Fig. 19.

Fig. 20.

Basire Sc.

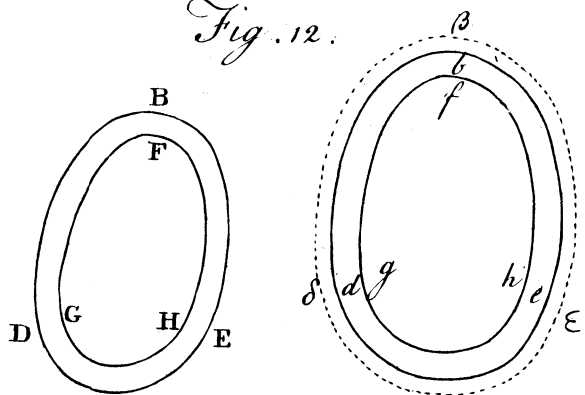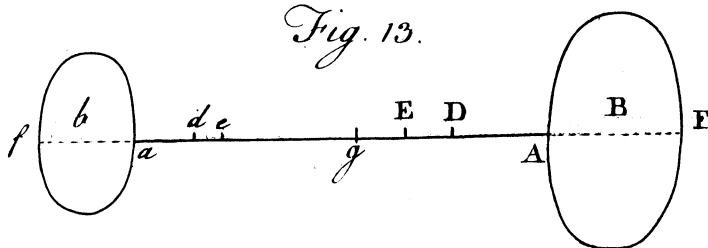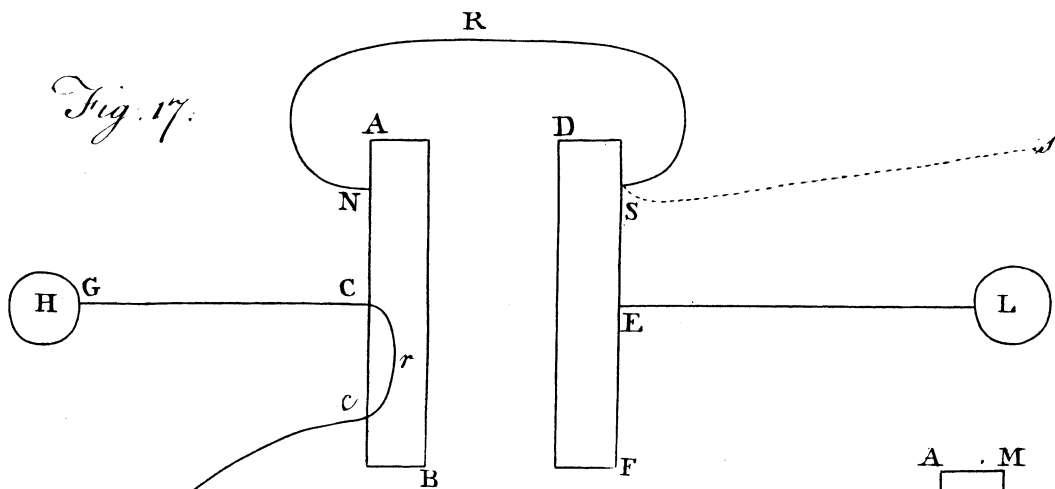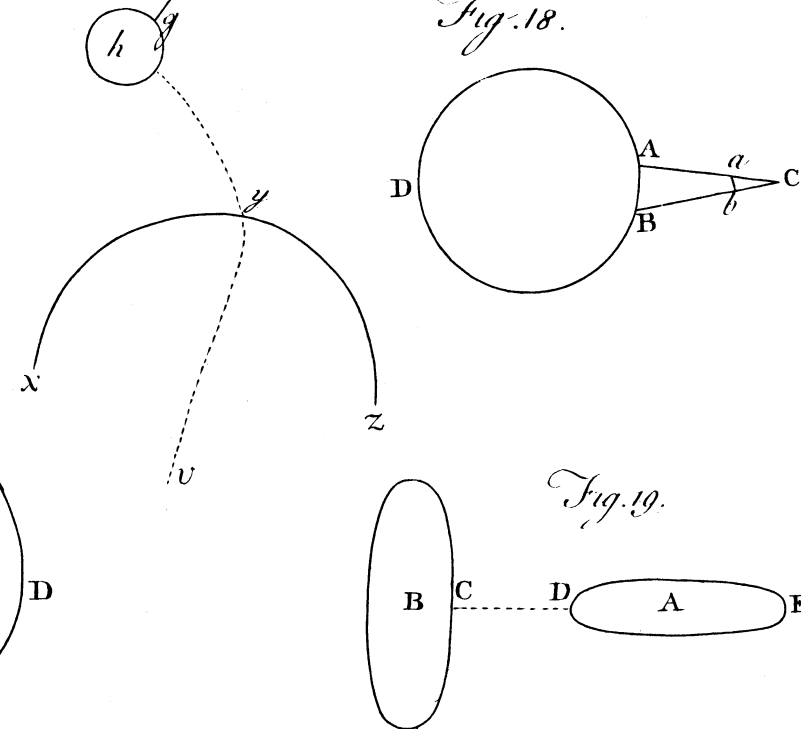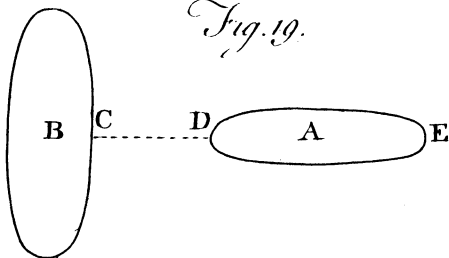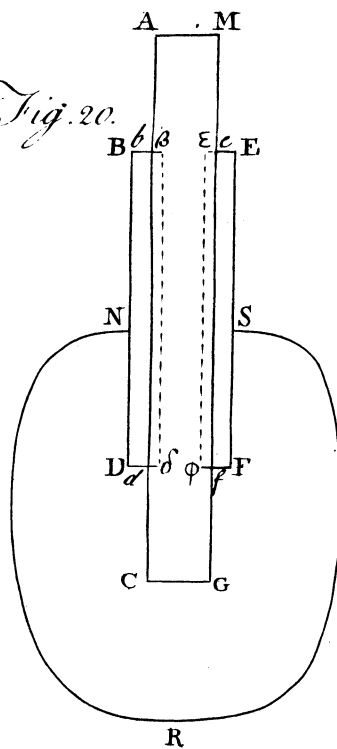charged, or both undercharged, but in different degrees; and let the redundance or deficience of fluid in each be very small in respect of the whole quantity of fluid in them : it is impossible for the fluid to be disposed accurately in a similar manner in both of them *; as it has been shewn that there will be a space, close to the surface, which will either be as full of fluid as it can hold, or will be intirely deprived of fluid; but it will be disposed as nearly in a similar manner in both, as is possible. To explain this, let B D E and *b d e* (fig. 12) be the two similar bodies; and let the space comprehended between the surfaces B D E and F G H (or the space B F as I shall call it for shortness) be that part of B D E, which is either as full of fluid as it can hold, or intirely deprived of it: draw the surface *f g b*, such that the space *b f*, shall be to the space B F, as the quantity of redundant or deficient fluid in *b d e*, to that in B D E, and that the thickness of the space *b f* shall every where bear the same proportion to the corre-sponding thickness of B F : then will the space *b f* be either as full of fluid as it can hold, or

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* By the fluid being disposed in a similar manner in both bodies, I mean that the quantity of redundant or deficient fluid in any small part of one body, is to that in the corresponding small part of the other, as the whole quantity of redundant or deficient fluid in one body, to that in the other. By the quan-tity of deficient fluid in a body, I mean the quantity of fluid wanting to saturate it. Notwithstanding the impropriety of this expression, I must beg leave to make use of it, as it will fre-quently save a great deal of circumlocution.

intirely deprived of it; and the fluid within the space fgh will be difposed very nearly fimilarly to that in the fpace F G H.

For it is plain, that if the fluid could be difpofed accurately in a fimilar manner in both bodies, the fluid would be in equilibrio in one body, if it was in the other: therefore draw the furface $\beta\delta\epsilon$, fuch that the thicknefs of the fpace βf fhall be every where to the correfponding thicknefs of B F, as the diameter of bde to the diameter of BDE; and let the redundant fluid or matter in bf be fpread uniformly over the fpace βf; then if the fluid in the fpace fgh is difpofed exactly fimilarly to that in F G H, it will be in equilibrio; as the fluid will then be difpofed exactly fimilarly in the fpaces $\beta\delta\epsilon$ and B D E: but as by the fuppofition, the thicknefs of the fpace βf is very fmall in refpect of the diameter of bde, the fluid or matter in the fpace bf will exert very nearly the fame force on the reft of the fluid, whether it is fpread over the fpace βf, or whether it is collected in bf.

P R O P. XVIII.

Let two bodies, B and b, be connected to each other by a canal of any kind, and be either over or undercharged: it is plain that the quantity of redundant or deficient fluid in B, would bear exactly the fame proportion to that in b, whatever fort of matter B confifted of, if it was poffible for the redundant or deficient fluid in

I   any

any body, to be difpofed accurately in the fame manner, whatever fort of matter it confifted of. For fuppofe B to confift of any fort of matter; and let the fluid in the canal and two bodies be in equilibrio: let now B be made to confift of fome other fort of matter, which requires a different quantity of fluid to faturate it; but let the quantity and difpofition of the redundant or deficient fluid in it remain the fame as before: it is plain that the fluid will ftill be in equilibrio; as the attraction or repulfion of any body depends only on the quantity and difpofition of the redundant and deficient fluid in it. Therefore, by the preceeding propofition, the quantity of redundant or deficient fluid in B, will actually bear very nearly the fame proportion to that in b, whatever fort of matter B confifts of; provided the quantity of redundant or deficient fluid in it is very fmall in refpect of the whole.

P R O P. XIX.

Let two bodies B and b (fig. 11.) be connected together by a very flender canal A D d a, either ftraight or crooked: let the canal be everywhere of the fame breadth and thicknefs; fo that all fections of this canal made by planes perpendicular to the direction of the canal in that part, fhall be equal and fimilar: let the canal be compofed of uniform matter; and let the electric fluid therein be fuppofed incompreffible, and of fuch denfity as exactly to faturate the matter

therein ; and let it, neverthelefs, be able to move readily along the canal ; and let each particle of fluid in the canal be attracted and repelled by the matter and fluid in the canal and in the bodies B and b, juft in the fame manner that it would be if it was not incompreffible * ; and let the bodies B and b be either over or undercharged. I fay that the force with which the whole quantity of fluid in the canal is impelled from A towards D, in the direction of the axis of the canal, by the united attractions and repulfions of the two bodies, muft be nothing ; as otherwife the fluid in the canal could not be at reft : obferving that by the force with which the whole quantity of fluid is impelled in the direction of the axis of the canal, I mean the fum of the forces, with which the fluid in each part of the canal is impelled in the direction of the axis of the canal in that place, from A towards D ; and obferving alfo, that an impulfe in the contrary direction from D towards A muft be looked upon as negative.

For as the canal is exactly faturated with fluid, the fluid therein is attracted or repelled only by the redundant matter or fluid in the two bodies. Suppofe now that the fluid in any fection of the canal, as E e,

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* This fuppofition of the fluid in the canal being incompreffible, is not mentioned as a thing which can ever take place in nature, but is merely imaginary ; the reafon for making of which will be given hereafter.

is impelled with any given force in the direction of
the canal at that place, the section D d would, in
consequence thereof, be impelled with exactly the
same force in the direction of the canal at D, if the
fluid between E e and D d was not at all attracted or
repelled by the two bodies; and, consequently, the
section D d is impelled in the direction of the canal,
with the sum of the forces, with which the fluid in
each part of the canal is impelled, by the attraction
or repulsion of the two bodies in the direction of the
axis in that part; and consequently, unless this sum
was nothing, the fluid in D d could not be at rest.

COROLLARY.

Therefore, the force with which the fluid in the
canal is impelled one way in the direction of the axis,
by the body B, must be equal to that with which it
impelled by b in the contrary direction.

PROP. XX.

Let two similar bodies B and b (fig. 13.) be connected
by the very slender cylindric or prismatic ca-
nal A a, filled with incompressible fluid, in the
same manner as described in the preceding pro-
position: let the bodies be overcharged; but
let the quantity of redundant fluid in each bear
so small a proportion to the whole, that the
fluid may be considered as disposed in a similar
manner in both; let the bodies also be similarly
situated in respect of the canal A a; and let
them be placed at an infinite distance from each

4 L 2　　　　　other,

other, or at fo great an one, that the repulfion
of either body on the fluid in the canal fhall
not be fenfibly lefs than if they were at an in-
finite diftance : then, if the electric attraction
and repulfion is inverfely as the n power of the
diftance, n being greater than one, and lefs
than three, the quantity of redundant fluid in
the two bodies will be to each other, as the
$n - 1$ power of their correfponding diameters
A F and af.

For if the quantity of redundant fluid in the two
bodies is in this proportion, the repulfion of one
body on the fluid in the canal, will be equal to that
of the other body on it in the contrary direction;
and, confequently, the fluid will have no tendency
to flow from one body to the other, as may thus be
proved. Take the points D and E very near to each
other; and take da to D A, and ea to E A, as cf to
A F; the repulfion of the body B on a particle at D,
will be to the repulfion of b on a particle at d, as
$\frac{1}{AF}$ to $\frac{1}{af}$; for, as the fluid is difpofed fimilarly in
both bodies, the quantity of fluid in any fmall part
of B, is to the quantity in the correfponding part of
b, as AF^{n-1} to af^{n-1}; and, confequently, the re-
pulfion of that fmall part of B, on D, is to the
repulfion of the correfponding part of b, on d, as
$\frac{AF^{n-1}}{AF^n}$, or $\frac{1}{AF}$, to $\frac{1}{af}$. But the quantity of fluid in
the fmall part D E of the canal, is to that in de, as
D E to de, or as AF to af; therefore the repulfion
of

of B on the fluid in D E, is equal to that of b on the fluid in de : therefore, taking ag to A a, as af to A F, the repulsion of b on the fluid in ag, is equal to that of B on the fluid in A a; but the repulsion of b on ag may be considered as the same as its repulsion on A a; for, by the supposition, the repulsion of B on Aa may be considered as the same as if it was continued infinitely ; and therefore, the repulsion of b on ag may be considered as the same as if it was continued infinitely.

N. B. If n was not greater than one, it would be impossible for the length of A a to be so great, that the repulsion of B on it might be considered as the same as if it was continued infinitely; which was my reason for requiring n to be greater than one.

COROLLARY.

By just the same method of reasoning it appears, that if the bodies are undercharged, the quantity of deficient fluid in b will be to that in B, as af^{n-1} to A F^{n-1}.

PROP. XXI.

Let a thin flat plate be connected to any other body, as in the preceding proposition, by a canal of incompressible fluid, perpendicular to the plane of the plate; and let that body be overcharged, the quantity of redundant fluid in the plate will bear very nearly the same

3   proportion

proportion to that in the other body, whatever the thicknefs of the plate may be, provided its thicknefs is very fmall in proportion to its breadth, or fmalleft diameter.

For there can be no doubt, but what, under that reftriction, the fluid will be difpofed very nearly in the fame manner in the plate, whatever its thicknefs may be; and therefore its repulfion on the fluid in the canal will be very nearly the fame, whatever its thicknefs may be.

P R O P. XXII.

Let A B and D F (fig. 14.) reprefent two equal and parallel circular plates, whofe centers are C and E; let the plates be placed fo, that a right line joining their centers fhall be perpendicular to the plates; let the thicknefs of the plates be very fmall, in refpect of their diftance C E; let the plate A B communicate with the body H, and the plate D F with the body L, by the canals C G and E M of incompreffible fluid, fuch as are defcribed in Prop. XIX; let thefe canals meet their refpective plates in their centers C and E, and be perpendicular to the plane of the plates; and let their length be fo great, that the repulfion of the plates on the fluid in them may be confidered as the fame, as if they were continued infinitely; let the body H be overcharged, and let L be faturated. It is plain, from Prop. XII. that D F will be undercharged, and A B will be more over-
charged

charged than it would otherwife be. Suppofe, now, that the redundant fluid in A B is dif-pofed in the fame manner as the deficient fluid is in D F; let P be to one as the force with which the plate A B would repel the fluid in C E, if the canal M E was continued to C, is to the force with which it would repel the fluid in C M; and let the force with which A B re-pels the fluid in C G, be to the force with which it would repel it, if the redundant fluid in it was fpread uniformly, as π to 1; and let the force with which the body H repels the fluid in C G, be the fame with which a quan-tity of redundant fluid, which we will call B, fpread uniformly over A B, would repel it in the contrary direction. Then will the redun-dant fluid in A B be equal to $\dfrac{B}{2 P \pi - P^2 \pi}$, and therefore, if P is very fmall, will be very nearly equal to $\dfrac{B}{2 P \pi}$; and the deficient fluid in D F will be to the redundant fluid in A B, as $1 - P$ to one, and therefore, if P is very fmall, will be very nearly equal to the redundant fluid in A B.

For it is plain, that the force with which A B repels the fluid in E M, muft be equal to that with which D F attracts it; for otherwife, fome fluid would run out of D F into L, or out of L into D F: for the fame reafon, the excefs of the repulfion of A B on the fluid in C G, above the attraction of F D thereon, muft be equal to the force with which a quantity

quantity of redundant fluid equal to B, ſpread uni-
formly over A B, would repel it, or it muſt be
equal to that with which a quantity equal to $\frac{B}{\pi}$,
ſpread in the manner in which the redundant fluid
is actually ſpread in A B, would repel it. By the
ſuppoſition, the force with which A B repels the
fluid in E M, is to the force with which it would
repel the fluid in C M, ſuppoſing E M to be con-
tinued to C, as 1 — P to one; but the force with
which any quantity of fluid in A B would repel the
fluid in CM, is the ſame with which an equal quan-
tity ſimilarly diſpoſed in D F, would repel the fluid
in E M ; therefore, the force with which the re-
dundant fluid in A B repels the fluid in E M, is to
that with which an equal quantity ſimilarly diſpoſed
in D F, would repel it, as 1 — P to one : therefore,
if the redundant fluid in A B be called A, the defi-
cient fluid in D F muſt be A × 1 — P: for the ſame
reaſon, the force with which D F attracts the fluid
in C G, is to that with which A B repels it, as
A × 1 — P × 1 — P, or A × $\overline{1 - P}^{2}$, to A ; there-
fore, the exceſs of the force with which A B repels
C G above that with which D F attracts it, is equal
to that with which a quantity of redundant fluid
equal to A — $\overline{A \times 1 - P}^{2}$, or $\overline{A \times 2 P - P^{2}}$, ſpread
over A B, in the manner in which the redundant
fluid therein is actually ſpread, would repel it : there-
fore, $\overline{A \times 2 P - P^{2}}$ muſt be equal to $\frac{B}{\pi}$, or A muſt
be equal to $\frac{B}{2 P \pi - P^{2} \pi}$.

COROL.

COROL. I.

If the denſity of the redundant fluid near the middle of the plate A B, is leſs than the mean denſity, or the denſity which it would everywhere be of, if it was ſpread uniformly, in the ratio of δ to one; and if the diſtance of the two plates is ſo ſmall, that $E C^{n-1}$ is very ſmall in reſpect of $A C^{n-1}$, and that $E C^{3-n}$ is very ſmall in reſpect of $A C^{3-n}$, the quantity of redundant fluid in A B will be greater than $\frac{B}{2} \times \overline{\frac{AC}{EC}}\Big|^{3-n}$, and leſs than $\frac{B}{2\delta} \times \overline{\frac{AC}{EC}}\Big|^{3-n}$, but will approach much nearer to the latter value than the former. For, in this caſe, $P\pi$ is, by Lemma X. Corol. IV. leſs than $\overline{\frac{EC}{AC}}\Big|^{3-n}$, and greater than $\overline{\frac{EC}{AC}}\Big|^{3-n} \times \delta$, but approaches much nearer to the latter value than the former; and if $E C^{3-n}$ is very ſmall in reſpect of $A C^{3-n}$, P is very ſmall.

REMARKS.

If D F was not undercharged, it is certain that A B would be conſiderably more overcharged near the circumference of the circle than near the center; for if the fluid was ſpread uniformly, a particle placed anywhere at a diſtance from the center, as at N, would be repelled with conſiderably more force towards the circumference than it would towards the

center. If the plates are very near together, and, confequently, D F nearly as much undercharged as A B is overcharged, A B will ftill be more over-charged near the circumference than near the center, but the difference will not be near fo great as in the former cafe : for, let N R be many times greater than C E, and N S lefs than C E; and take E r and E s equal to C R and C S, there can be no doubt, I think, but that the deficient fluid in D F will be lodged nearly in the fame manner as the redundant fluid in A B; and therefore, the repulfion of the redundant fluid at R, on a particle at N, will be very nearly balanced by the attraction of the redundant matter at r, for R is not much nearer to N than r is; but the re-pulfion of S will not be near balanced by that of s; for the diftance of S from N is much lefs than that of s. Let now a fmall circle, whofe diameter is S T, be drawn round the center N, on the plane of the plate; as the denfity of the fluid is greater at T than at S, the repulfion of the redundant fluid within the fmall circle tends to impel the point N towards C; but as there is a much greater quantity of fluid between N and B, than between N and A, the re-pulfion of the fluid without the fmall circle tends to balance that; but the effect of the fluid within the fmall circle is not much lefs than it would be, if D F was not undercharged; whereas much the greater part of the effect of that part of the plate on the out-fide of the circle, is taken off by the effect of the correfponding part of D F : confequently, the dif-ference of denfity between T and S will not be near fo great, as if D F was not undercharged. Hence I fhould imagine, that if the two plates are

very

very near together, the denfity of the redundant fluid near the center will not be much lefs than the mean denfity, or δ will not be much lefs than one ; moreover, the lefs the diftance of the plates, the nearer will δ approach to one.

Corol. II.

Let now the body H confift of a circular plate, of the fame fize as A B, placed fo, that the canal C G fhall pafs through its center, and be perpendicular to its plane; by the fuppofition, the force with which H repels the fluid in the canal C G, is the fame with which a quantity of fluid, equal to B, fpread uniformly over A B, would repel it in the contrary direction: therefore, if the fluid in the plate H was fpread uniformly, the quantity of redundant fluid therein would be B, and if it was all collected in the circumference, would be $\dfrac{2\,\text{B}}{3-n}$; and therefore the real quantity will be greater than B, and lefs than $\dfrac{2\,\text{B}}{3-n}$.

Corol. III.

Therefore, if we fuppofe δ to be equal to one, the quantity of redundant fluid in A B will exceed that in the plate H, in a greater ratio than that of $\overline{\dfrac{\text{A C}}{\text{C E}}}^{\,3-n} \times \dfrac{3-n}{4}$ to one, and lefs than that of $\overline{\dfrac{\text{A C}}{\text{C E}}}^{\,3-n} \times \dfrac{1}{2}$ to one ; and from the preceding remarks it appears, that the real quantity of redundant fluid in A B can

hardly

hardly be much greater than it would if δ was equa
to one.

Corol. IV.

Hence, if the electric attraction and repulsion is
inversely as the square of the distance, the redundant
fluid in A B, supposing δ to be equal to one, will
exceed that in the plate H, in a greater ratio than
that of A C to 4 C E, and less than that of A C to
2 C E.

Corol. V.

Let now the body H consist of a globe, whose
diameter equals A B; the globe being situated in
such a manner, that the canal C G, if continued,
would pass through its center ; and let the electric
attraction and repulsion be inversely as the square of
the distance, the quantity of redundant fluid in the
globe will be 2 B : for the fluid will be spread uni-
formly over the surface of the globe, and its repul-
sion on the canal will be the same as if it was all
collected in the center of the sphere, and will there-
fore be the same with which an equal quantity, dif-
posed in the circumference of A B, would repel it in
the contrary direction, or with which half that quan-
tity, or B, would repel it, if spread uniformly over
the plate.

COROL. VI.

Therefore, if δ was equal to one, the redundant fluid in A B would exceed that in the globe, in the ratio of A C to 4 C E; and therefore, it will in reality exceed that in the globe, in a rather greater ratio than that of A C to 4 C E; but if the plates are very near together, it will approach very near thereto, and the nearer the plates are, the nearer it will approach thereto.

COROL. VII.

Whether the electric repulsion is inversely as the square of the distance or not, if the body H is as much undercharged, as it was before overcharged, A B will be as much undercharged as it was before overcharged, and D F as much overcharged as it was before undercharged.

COROL. VIII.

If the size and distance of the plates be altered, the quantity of redundant or deficient fluid in the body H remaining the same, it appears, by comparing this proposition with the 20th and 21st propositions, that the quantity of redundant and deficient fluid in A B will be as $A C^{n-1} \times \overline{\dfrac{A C}{E C}}^{3-n}$, or as $\dfrac{A C^2}{E C^{3-n}}$, supposing the value of δ to remain the same.

PROP.

P R O P. XXIII.

Let A E (fig. 15.) be a cylindric canal, infinitely
continued beyond E ; and let A F be a bent
canal, meeting the other at A, and infinitely
continued beyond F : ſet the ſection of this ca-
nal, in all parts of it, be equal to that of the
cylindric canal, and let both canals be filled
with uniform fluid of the ſame denſity : the
force with which a particle of fluid P, placed
anywhere at pleaſure, repels the whole quan-
tity of fluid in A F, in the direction of the ca-
nal, is the ſame with which it repels the fluid
in the canal A E, in the direction A E.

On the center P, draw two circular arches B D
and $b\,d$, infinitely near to each other, cutting A E in
B and β, and A F in D and δ, and draw the radii
P b and P d.   As P B $=$ P D, the force with which
P repels a particle at B, in the direction B β, is to
that with which it repels an equal particle at D, in
the direction D δ, as $\dfrac{\mathrm{B}\,b}{\mathrm{B}\,\beta}$ to $\dfrac{\mathrm{D}\,d}{\mathrm{D}\,\delta}$, or as $\dfrac{1}{\mathrm{B}\,\beta}$ to $\dfrac{1}{\mathrm{D}\,\delta}$; and
therefore, the force with which it repels the whole
fluid in B β, in the direction B β, is the ſame with
which it repels the whole fluid in D δ, in the direc-
tion D δ, that is in the direction of the canal ; and
therefore, the force with which it repels the whole
fluid in A E, in the direction A E, is the ſame with
which it repels the whole fluid in A F, in the direc-
tion of the canal.

COROLLARY.

If the bent canal ADF, inftead of being infinitely continued, meets the cylindric canal in E, as in fig. 16. the repulfion of P on the fluid in the bent canal A D E, in the direction of the canal, will ftill be equal to its repulfion on that in the cylindric canal A E, in the direction A E.

PROP. XXIV.

If two bodies, for inftance the plate A B, and the body H, of Prop. XXII. communicate with each other, by a canal filled with incompreffible fluid, and are either over or undercharged, the quantity of redundant fluid in them will bear the fame proportion to each other, whether the canal by which they communicate is ftraight or crooked, or into whatever part of the bodies the canal is inferted, or in whatever manner the two bodies are fituated in refpect of each other; provided that their diftance is infinite, or fo great that the repulfion of each body on the fluid in the canal fhall not be fenfibly lefs than if it was infinite.

Let the parellelograms A B and D F (fig. 17.) reprefent the two plates, and H and L the bodies communicating with them : let now H be removed to *h*; and let it communicate with A B, by the bent canal *g c*; the quantity of fluid in the plates and
bodies

bodies remaining the fame as before; and let us, for the fake of eafe in the demonftration, fuppofe the canal $g c$ to be every where of the fame thicknefs as the canal G C; though the propofition will evidently hold good equally, whether it is or not: the fluid will ftill be in equilibrio. For let us firft fuppofe the canal $g c$ to be continued through the fubftance of the plate A B, to C, along the line $c r$ C; the part $c r$ C being of the fame thicknefs as the reft of the canal, and the fluid in it of the fame denfity: by the preceding propofition, the repulfion or attraction of each particle of fluid or matter in the plates A B and D F, on the fluid in the whole canal C $r c g$, in the direction of that canal, is equal to its repulfion or attraction on the fluid in the canal C G, in the direction C G; and therefore the whole repulfion or attraction of the two plates on the canal C $r c g$, is equal to their repulfion or attraction on C G: but as the fluid in the plate A B is in equilibrio, each particle of fluid in the part C $r c$ of the canal, is impelled by the plates, with as much force in one direction as the other; and confequently the plates impel the fluid in the canal $c g$, with as much force as they do that in the whole canal C $r c g$, that is, with the fame force that they impel the fluid in C G. In like manner the body h impels the fluid in $c g$, with the fame force that H does the fluid in C G; and confequently h impels the fluid in $c g$, one way in the direction of the canal, with the fame force that the two plates impel it the contrary way; and therefore the fluid in $c g$ has no tendency to flow from one body to the other.

COROL-

COROLLARY.

By the fame method of reafoning, with the help of the corollary to the 23d propofition, it appears, that if A B and H each communicate with a third body, by canals of incompreffible fluid, and a communication is made between A B and H by another canal of incompreffible fluid, the fluid will have no tendency to flow from one to the other through this canal; fuppofing that the fluid was in equilibrio before this communication was made. In like manner if A B and H communicate with each other, or each communicate with a third body, by canals of real fluid, inftead of the imaginary canals of incompreffible fluid ufed in thefe propofitions, and a communication is alfo made between them by a canal of incompreffible fluid, the fluid can have no tendency to flow from one to the other. The truth of the latter part of this corollary will appear by fuppofing an imaginary canal of incompreffible fluid to be continued through the whole length of the real one.

PROP. XXV.

Let now a communication be made between the two plates A B and D F, by the canal N R S of incompreffible fluid, of any length; and let the body H and the plate A B be overcharged. It is plain that the fluid will flow through that canal from A B to D F. Now the whole force with which the fluid in the canal is impelled

along it, by the joint action of the two plates, is the fame with which the whole quantity of fluid in the canal C G or *c g* is impelled by them; fuppofing the canal N R S to be every where of the fame breadth and thicknefs as C G or *c g*.

For fuppofe that the canal N R S, inftead of communicating with the plate D F, is bent back juft before it touches it, and continued infinitely along the line S *s*; the force with which the two plates impel the fluid in S *s*, is the fame with which they impel that in E L, fuppofing S *s* to be of the fame breadth and thicknefs as E L; and is therefore nothing; therefore the force with which they impel the fluid in N R S, is the fame with which they impel that in N R S *s*; which is the fame with which they impel that in C G.

P R O P. XXVI.

Let now *x y z* be a body of an infinite fize, containing juft fluid enough to faturate it; and let a communication be made between *h* and *x y z*, by the canal *h y* of incompreffible fluid, of the fame breadth and thicknefs as *g c* or G C; the fluid will flow through it from *h* to *x y z*; and the force with which the fluid in that canal is impelled along it, is equal to that with which the fluid in N R S is impelled by the two plates.

If

If the canal hy is of fo great a length, that the repulfion of h thereon is the fame as if it was continued infinitely, then the thing is evident : but if it is not, let the canal hy, inftead of communicating with xyz, fo that the fluid can flow out of the canal into xyz, be continued infinitely through its fubftance, along the line yv : now it muft be obferved that a fmall part of the body xyz, namely, that which is turned towards h, will by the action of h upon it, be rendered undercharged ; but all the reft of the body will be faturated ; for the fluid driven out of the undercharged part will not make the re· mainder, which is fuppofed to be of an infinite fize, fenfibly overcharged : now the force with which the fluid in the infinite canal hyv, is impelled by the body h and the undercharged part of xyz, is the fame with which the fluid in gc is impelled by them; but as the fluid in all parts of xyz is in equilibrio, a particle in any part of yv cannot be impelled in any direction ; and therefore the fluid in hy is impelled with as much force as that in hyv ; and therefore the fluid in hy is impelled with as much force as that in gc ; and is therefore impelled with as much force as the fluid in N R S is impelled by the two plates.

It perhaps may be afked, whether this method of demonftration would not equally tend to prove that the fluid in hy was impelled with the fame force as that in N R S, though xyz did not contain juft fluid enough to faturate it. I anfwer not; for this demonftration depends on the canal yv being continued, within the body xyz, to an infinite diftance beyond any over or undercharged part; which could

not

not be if $x\,y\,z$ contained either more or lefs fluid than that.

P R O P. XXVII.

Let two bodies B and b (fig. 13.) be joined by a cylindric or prifmatic canal A a, filled with real fluid; and not by an imaginary canal of incompreffible fluid as in the 20th propofition; and let the fluid therein be in equilibrio: the force with which the whole or any given part of the fluid in the canal, is impelled in the direction of its axis, by the united repulfions and attractions of the redundant fluid or matter in the two bodies and the canal, muft be nothing; or the force with which it is impelled one way in the direction of the axis of the canal, muft be equal to that with which it is impelled the other way.

For as the canal is fuppofed cylindric or prifmatic, no particle of fluid therein can be prevented from moving in the direction of the axis of it, by the fides of the canal; and therefore the force with which each particle is impelled either way in the direction of the axis, by the united attractions and repulfions of the two bodies and the canal, muft be nothing, otherwife it could not be at reft; and therefore the force with which the whole, or any given part of the fluid in the canal, is impelled in the direction of the axis, muft be nothing.

3 COROL.

COROL. I.

If the fluid in the canal is difpofed in fuch manner, that the repulfion or attraction of the redundant fluid or matter in it, on the whole or any given part of the fluid in the canal, has no tendency to impel it either way in the direction of the axis; then the force with which that whole or given part is impelled by the two bodies muft be nothing; or the force with which it is impelled one way in the direction of the axis, by the body B, muft be equal to that with which it is impelled in the contrary direction by the other body; but not if the fluid in the canal is difpofed in a different manner.

COROL. II.

If the bodies, and confequently the canal, is overcharged; then, in whatever manner the fluid in the canal is difpofed, the force with which the whole quantity of redundant fluid in the canal is repelled by the body B in the direction A *a*, muft be equal to that with which it is repelled by *b* in the contrary direction. For the force with which the redundant fluid is impelled in the direction A *a* by its own repulfion, is nothing; for the repulfion of the particles of any body on each other have no tendency to make the whole body move in any direction.

RE-

REMARKS.

When I firſt thought of the 20th and 22d pro-
poſitions, I imagined that when two bodies were
connected by a cylindric canal of real fluid, the re-
pulſion of one body on the whole quantity of fluid in
the canal, in one direction, would be equal to that of
the other body on it in the contrary direction, in
whatever manner the fluid was diſpoſed in the canal ;
and that therefore thoſe propoſitions would have held
good very nearly, though the bodies were joined by
cylindric canals of real fluid ; provided the bodies
were ſo little over or undercharged, that the quantity
of redundant or deficient fluid in the canal ſhould be
very ſmall in reſpect of the quantity required to ſa-
turate it ; and conſequently that the fluid therein
ſhould be very nearly of the ſame denſity in all parts.
But from the foregoing propoſition it appears that I
was miſtaken, and that the repulſion of one body on
the fluid in the canal is not equal to that of the
other body on it, unleſs the fluid in the canal is diſ-
poſed in a particular manner : beſides that, when two
bodies are both joined by a real canal, the attraction
or repulſion of the redundant matter or fluid in the
canal, has ſome tendency to alter the diſpoſition of
the fluid in the two bodies ; and in the 22d propo-
ſition, the canal C G exerts alſo ſome attraction or
repulſion on the canal E M : on all which accounts
the demonſtration of thoſe propoſitions is de-
fective, when the bodies are joined by real canals.
I have good reaſon however to think, that thoſe propo-
ſitions actually hold good very nearly when the bodies

are

are joined by real canals; and that, whether the canals are straight or crooked, or in whatever direction the bodies are situated in respect of each other: though I am by no means able to prove that they do: I therefore chose still to retain those propositions, but to demonstrate them on this ideal supposition, in which they are certainly true, in hopes that some more skilful mathematician may be able to shew whether they really hold good or not.

What principally makes me think that this is the case, is that as far as I can judge from some experiments I have made, the quantity of fluid in different bodies agrees very well with those propositions, on a supposition that the electric repulsion is inversely as the square of the distance. It should also seem from those experiments, that the quantity of redundant or deficient fluid in two bodies, bore very nearly the same proportion to each other, whatever is the shape of the canal by which they are joined, or in whatever direction they are situated in respect of each other.

Though the above propositions should be found not to hold good, when the bodies are joined by real canals, still it is evident, that in the 22d proposition, if the plates A B and D F are very near together, the quantity of redundant fluid in the plate A B will be many times greater than that in the body H, supposing H to consist of a circular plate of the same size as A B, and D F will be near as much undercharged as A B is overcharged.

Sir Isaac Newton supposes that air consists of particles which repel each other with a force inversely as the distance: but it appears plainly from the foregoing pages, that if the repulsion of the particles was

im

in this ratio; and extended indefinitely to all
diftances, they would compofe a fluid extremely
different from common air. If the repulfion of the
particles was inverfely as the diftance, but extended
only to a given very fmall diftance from their centers,
they would compofe a fluid of the fame kind as air,
in refpect of elafticity, except that its denfity would
not be in proportion to its compreffion: if the diftance
to which the repulfion extends, though very fmall, is
yet many times greater than the diftance of the
particles from each other, it might be fhewn, that the
denfity of the fluid would be nearly as the fquare
root of the compreffion. If the repulfion of the
particles extended indefinitely, and was inverfely as
fome higher power of the diftance than the cube, the
denfity of the fluid would be as fome power of the
compreffion lefs than $\frac{1}{3}$. The only law of repulfion,
I can think of, which will agree with experiment, is
one which feems not very likely; namely, that the
particles repel each other with a force inverfely as the
diftance; but that, whether the denfity of the fluid is
great or fmall, the repulfion extends only to the
neareft particles: or, what comes to the fame thing,
that the diftance to which the repulfion extends, is
very fmall, and alfo is not fixed, but varies in pro-
portion to the diftance of the particles.

PART II.

Containing a comparifon of the foregoing theory with experiment.

§ 1. It appears from experiment, that fome bodies fuffer the electric fluid to pafs with great readi-nefs between their pores; while others will not fuffer it to do fo without great difficulty; and fome hardly fuffer it to do fo at all. The firft fort of bodies are called conductors, the others non-con-ductors. What this difference in bodies is owing to I do not pretend to explain.

It is evident that the electric fluid in non-con-ductors may be confidered as moveable, or anfwers to the definition given of that term in p. 588. As to the fluid contained in non-conducting fubftances, though it does not abfolutely anfwer to the defi-nition of immoveable, as it is not abfolutely con-fined from moving, but only does fo with great difficulty; yet it may in moft cafes be looked upon as fuch without fenfible error.

Air does in fome meafure permit the electric fluid to pafs through it; though, if it is dry, it lets it pafs but very flowly, and not without diffi-culty; it is therefore to be called a non-conductor.

It appears that conductors would readily fuffer the fluid to run in and out of them, were it not for the air which furrounds them: for if the end of a conductor is inferted into a vacuum, the fluid runs in and out of it with perfect readinefs; but

when it is furrounded on all fides by the air, as no fluid can run out of it without running into the air, the fluid will not do fo without difficulty.

If any body is furrounded on all fides by the air, or other non-conducting fubftances, it is faid to be infulated: if on the other hand it any where communicates with any conducting body, it is faid to be not infulated. When I fay that a body communicates with the ground, or any other body, I would be underftood to mean that it does fo by fome conducting fubftance.

Though the terms pofitively and negatively electrified are much ufed, yet the precife fenfe in which they are to be underftood, feems not well afcertained; namely, whether they are to be underftood in the fame fenfe in which I have ufed the words over or undercharged, or whether, when any number of bodies, infulated and communicating with each other by conducting fubftances, are electrified by means of excited glafs, they are all to be called pofitively electrified (fuppofing, according to the ufual opinion, that excited glafs contains more than its natural quantity of electricity); even though fome of them, by the approach of a ftronger electrified body, are made undercharged. I fhall ufe the words in the latter fenfe; but as it will be proper to afcertain the fenfe in which I fhall ufe them more accurately, I fhall give the following definition.

In order to judge whether any body, as A, is pofitively or negatively electrified: fuppofe another body B, of a given fhape and fize, to be placed at an infinite diftance from it, and from any other

over

over or undercharged body ; and let B contain the fame quantity of electric fluid, as if it communicated with A by a canal of incompreffible fluid : then, if B is overcharged, I call A pofitively electrified ; and if it is undercharged, I call A negatively electrified ; and the greater the degree in which B is over or undercharged, the greater is the degree in which A is pofitively or negatively electrified.

It appears from the corollary to the 24th propofition, that if feveral bodies are infulated, and connected together by conducting fubftances, and one of thefe bodies is pofitively or negatively electrified, all the other bodies muft be electrified in the fame degree : for fuppofing a given body B to be placed at an infinite diftance from any over or undercharged body, and to contain the fame quantity of fluid as if it communicated with one of thofe bodies by a canal of incompreffible fluid, all the reft of thofe bodies muft by that corollary contain the fame quantity of fluid as if they communicated with B by canals of incompreffible fluid : but yet it is poffible that fome of thofe bodies may be overcharged, and others undercharged : for fuppofe the bodies to be pofitively electrified, and let an overcharged body D be brought near one of them, that body will become undercharged, provided D is fufficiently overcharged ; and yet by the definition it will ftill be pofitively electrified in the fame degree as before.

Moreover, if feveral bodies are infulated and connected together by conducting fubftances, and one of thefe bodies is electrified by excited glafs, there can be no doubt, I think, but what they

will

will all be pofitively electrified; for if there is no other over or undercharged body placed near any of thefe bodies, the thing is evident; and though fome of thefe bodies may, by the approach of a fufficiently overcharged body, be rendered undercharged; yet I do not fee how it is poffible to prevent a body placed at an infinite diftance, and communicating with them by a canal of incompreffible fluid, from being overcharged.

In like manner if one of thefe bodies is electrified by excited fealing wax, they will all be negatively electrified.

It is impoffible for any body communicating with the ground to be either pofitively or negatively electrified : for the earth, taking the whole together, contains juft fluid enough to faturate it, and confifts in general of conducting fubftances ; and confequently though it is poffible for fmall parts of the furface of the earth to be rendered over or undercharged, by the approach of electrified clouds or other caufes; yet the bulk of the earth, and efpecially the interior parts, muft be faturated with electricity. Therefore affume any part of the earth which is itfelf faturated, and is at a great diftance from any over or undercharged part ; any body communicating with the ground, contains as much electricity as if it communicated with this part by a canal of incompreffible fluid, and therefore is not at all electrified.

If any body A, infulated and faturated with electricity, is placed at a great diftance from any over or undercharged body, it is plain that it cannot be electrified ; but if an overcharged body is
brought

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brought near it, it will be politively electrified; for suppofing A to communicate with any body B, at an infinite diftance, by a canal of incompreffible fluid, it is plain that unlefs B is overcharged, the fluid in the canal could not be in equilibrio, but would run from A to B. For the fame reafon a body infulated and faturated with fluid, will be negatively electrified if placed near an undercharged body.

§ 2. The phænomena of the attraction and repulfion of electrified bodies feem to agree exactly with the theory; as will appear by confidering the following cafes.

C A S E I. Let two bodies, A and B, both conductors of electricity, and both placed at a great diftance from any other electrified bodies, be brought near each other. Let A be infulated, and contain juft fluid enough to faturate it ; and let B be pofitively electrified. They will attract each other ; for as B is pofitively electrified, and at a great diftance from any overcharged body, it will be overcharged ; therefore, on approaching A and B to each other, fome fluid will be driven from that part of A which is neareft to B to the further part : but when the fluid in A was fpread uniformly, the repulfion of B on the fluid in A was equal to its attraction on the matter therein ; therefore, when fome fluid is removed from thofe parts where the repulfion of B is ftrongeft to thofe where it is weaker, B will repel the fluid in A with lefs force than it attracts the matter ; and confequently the bodies will attract each other.

C A S E

CASE II. If we now fuppofe that the fluid is at liberty to efcape from out of A, if it has any difpofition to do fo, the quantity of fluid in it before the approach of B being ftill fufficient to faturate it; that is, if A is not infulated and not electrified, B being ftill pofitively electrified, they will attract with more force than before: for in this cafe, not only fome fluid will be driven from that part of A which is neareft to B to the oppofite part, but alfo fome fluid will be driven out of A.

It muft be obferved, that if the repulfion of B on a particle at E, (fig. 19.) the fartheft part of A, is very fmall in refpect of its repulfion on an equal particle placed at D, the neareft part of A, the two bodies will attract with very nearly the fame force, whether A is infulated or not; but if the repulfion of B, on a particle at E, is very near as great as on one at D, they will attract with very little force if A is infulated. For inftance, let a fmall overcharged ball be brought near one end of a long conductor not electrified; they will attract with very near the fame force, whether the conductor be infulated or not; but if the conductor be overcharged, and brought near a fmall unelectrified ball, they will not attract with near fo much force, if the ball is infulated, as if it is not.

CASE III. If we now fuppofe that A is negatively electrified, and not infulated, it is plain that they will attract with more force than in the laft
<div align="right">cafe;</div>

cafe; as A will be ftill more undercharged in this cafe, than in the laft.

N. B. In thefe three cafes, we have not as yet taken notice of the effect which the body A will have in altering the quantity and difpofition of the fluid in B; but in reality this will make the bodies attract each other with more force than they would otherwife do; for in each of thefe cafes the body A attracts the fluid in B; which will caufe fome fluid to flow from the farther parts of B to the nearer, and will alfo caufe fome fluid to flow into it, if it is not infulated, and will confequently caufe B to act upon A with more force than it would otherwife do.

Case IV. Let us now fuppofe that B is negatively electrified; and let A be infulated, and contain juft fluid enough to faturate it; they will attract each other; for B will be undercharged; it will therefore attract the fluid in A, and will caufe fome fluid to flow from the fartheft part of A, where it is attracted with lefs force, to the nearer part, where it is attracted with more force; fo that B will attract the fluid in A with more force than it repels the matter.

Case V, and VI. If A is now fuppofed to be not infulated and not electrified, B being ftill negatively electrified, it is plain that they will attract with more force than in the laft cafe : and if A is pofitively electrified, they will attract with ftill more force.

In thefe three laft cafes alfo, the effect which A has in altering the quantity and difpofition of the
fluid

fluid in B, tends to increase the force with which the two bodies attract.

CASE VII. It is plain that a non-conducting body saturated with fluid, is not at all attracted or repelled by an over or undercharged body, until, by the action of the electrified body on it, it has either acquired some additional fluid from the air, or had some driven out of it, or till some fluid is driven from one part of the body to the other.

CASE VIII. Let us now suppose that the two bodies A and B are both positively electrified in the same degree. It is plain, that were it not for the action of one body on the other, they would both be overcharged, and would repel each other. But it may perhaps be said, that one of them as A may, by the action of the other on it, be either rendered undercharged on the whole, or at least may be rendered undercharged in that part nearest to B; and that the attraction of this undercharged part on a particle of the fluid in B, may be greater than the repulsion of the more distant overcharged part; so that on the whole the body A may attract a particle of fluid in B. If so, it must be affirmed that the body B repels the fluid in A; for otherwise, that part of A which is nearest to B could not be rendered undercharged. Therefore, to obviate this objection, let the bodies be joined by the straight canal DC of incompressible fluid (fig. 19.). The body B will repel the fluid in all parts of this canal; for as A is supposed to attract the fluid in B, B will not only be more overcharged than it would otherwise be, but it will also be more
over-

overcharged in that part neareſt to A than in the op-
poſite part. Moreover, as the near undercharged
part of A is ſuppoſed to attract a particle of fluid in
B with more force than the more diſtant overcharged
part repels it; it muſt, *a fortiori*, attract a particle
in the canal with more force than the other repels it;
therefore the body A muſt attract the fluid in the
canal; and conſequently ſome fluid muſt flow from
B to A, which is impoſſible; for as A and B are
both electrified in the ſame degree, they contain the
ſame quantity of fluid as if they both communicated
with a third body at an infinite diſtance, by canals of
incompreſſible fluid; and therefore, by the corollary
to Prop. 24, if a communication is made between
them by a canal of incompreſſible fluid, the fluid
would have no diſpoſition to flow from one to the
other.

CASE IX. But if one of the bodies as A is poſi-
tively electrified in a leſs degree than B, then it is
poſſible for the bodies to attract each other; for in
this caſe the force with which B repels the fluid in
A may be ſo great, as to make the body A either
intirely undercharged, or at leaſt to make the neareſt
part of it ſo much undercharged, that A ſhall on the
whole attract a particle of fluid in B.

It may be worth remarking with regard to this
caſe, that when two bodies, both electrified poſitively
but unequally, attract each other, you may by re-
moving them to a greater diſtance from each other,
cauſe them to repel; for as the ſtronger electrified
body repels the fluid in the weaker with leſs force
when removed to a greater diſtance, it will not be

able to drive fo much fluid out of it, or from the nearer to the further part, as when placed at a lefs diftance.

CASE X, and XI. By the fame reafoning it appears, that if the two bodies are both negatively electrified in the fame degree, they muft repel each other: but if they are both negatively electrified in different degrees, it is poffible for them to attract each other.

All thefe cafes are exactly conformable to experiment.

CASE XII. Let two cork balls be fufpended by conducting threads from the fame pofitively electrified body, in fuch manner that if they did not repel, they would hang clofe together: they will both be equally electrified, and will repel each other: let now an overcharged body, more ftrongly electrified than them, be brought under them; they will become lefs overcharged, and will feparate lefs than before: on bringing the body ftill nearer, they will become not at all overcharged, and will not feparate at all: and on bringing the body ftill nearer, they will become undercharged, and will feparate again.

CASE XIII. Let all the air of a room be overcharged, and let two cork balls be fufpended clofe to each other by conducting threads communicating with the wall. By Prop. 15, it is highly probable that the balls will be undercharged; and therefore they fhould repel each other.

Thefe

These two last cases are experiments of Mr. Canton's, and are described in Philof. Transf. 1753, p. 350, where are other experiments of the same kind, all readily explicable by the foregoing theory.

I have now considered all the principal or fundamental cases of electric attractions and repulsions which I can think of; all of which appear to agree perfectly with the theory.

§ 3. On the cases in which bodies receive electricity from or part with it to the air.

Lemma I.

Let the body A (fig. 6.) either stand near some over or undercharged body, or at a distance from any. It seems highly probable, that if any part of its surface, as M N, is overcharged, the fluid will endeavour to run out through that part, provided the air adjacent thereto is not overcharged.

For let G be any point in that surface, and P a point within the body, extremely near to it; it is plain that a particle of fluid at P, must be repelled with as much force in one direction as another (otherwise it could not be at rest) unless all the fluid between P and G is pressed close together, in which case it may be repelled with more force towards G than it is in the contrary direction : now a particle at G is repelled in the direction P G, *i. e.* from P to G, by all the redundant fluid between P and G; and a particle at P is repelled by the same fluid in the contrary direction; so that as the particle at P is repelled with not less force in the direction P G than in the

contrary,

contrary, I do not fee how a particle at G can help being repelled with more force in that direction than the contrary, unlefs the air on the outfide of the furface M N was more overcharged than the fpace between P and G.

In like manner, if any part of the furface is undercharged, the fluid will have a tendency to run in at that part from the air.

The truth of this is fomewhat confirmed by the third problem; as in all the cafes of that problem, the fluid was fhewn to have a tendency to run out of the fpaces A D and E H, at any furface which was overcharged, and to run in at any which was undercharged.

Corol. I.

If any body at a diftance from other over or undercharged bodies, be pofitively electrified, the fluid will gradually run out of it from all parts of its furface into the adjoining air ; as it is plain that all parts of the furface of that body will be overcharged : and if the body is negatively electrified, the fluid will gradually run into it at all parts of its furface from the adjoining air.

Corol. II.

Let the body A (fig. 6.) infulated and containing juft fluid enough to faturate it, be brought near the overcharged body B; that part of the furface of A which is turned towards B will by Prop. 11. be rendered

dered undercharged, and will therefore imbibe electricity from the air; and at the oppofite furface R S, the fluid will run out of the body into the air.

Corol. III.

If we now fuppofe that A is not infulated, but communicates with the ground, and confequently that it contained juft fluid enough to faturate it before the approach of B, it is plain that the furface M N will be more undercharged than before; and therefore the fluid will run in there with more force than before; but it can hardly have any difpofition to run out at the oppofite furface R S; for if the canal by which A communicates with the ground is placed oppofite to B, as in figure 5, then the fluid will run out through that canal till it has no longer any tendency to run out at R S; and by the remarks at the end of Prop. 27, it feems probable, that the fluid in A will be nearly in the fame quantity, and difpofed nearly in the fame manner, into whatever part of A the canal is inferted by which it communicates with the ground.

Corol. IV.

If B is undercharged the cafe will be reverfed; that is, it will run out where it before run in, and will run in where it before run out.

As far as I can judge, thefe corollaries feem conformable to experiment: thus far is certain, that bodies at a diftance from other electrified bodies receive

ceive electricity from the air, if negatively electrified, and part with some to it if positively electrified: and a body not electrified and not insulated receives electricity from the air if brought near an overcharged body, and loses some when brought near an undercharged body: and a body insulated and containing its natural quantity of fluid, in some cases, receives, and in others loses electricity, when brought near an over or undercharged body.

§ 4. The well-known effects of points in causing a quick discharge of electricity seem to agree very well with this theory.

It appears from the 20th proposition, that if two similar bodies of different sizes are placed at a very great distance from each other, and connected by a slender canal, and overcharged, the force with which a particle of fluid placed close to corresponding parts of their surface is repelled from them, is inversely as the corresponding diameters of the bodies. If the distance of the two bodies is small, there is not so much difference in the force with which the particle is repelled by the two bodies; but still, if the diameters of the two bodies are very different, the particle will be repelled with much more force from the smaller body than from the larger. It is true indeed that a particle placed at a certain distance from the smaller body, will be repelled with less force than if it be placed at the same distance from the greater body; but this distance is, I believe, in most cases pretty considerable; if the bodies are spherical, and the repulsion inversely as the square of the distance, a particle placed at any distance from the surface of the

6   smaller

smaller body less than a mean proportional between the radii of the two bodies, will be repelled from it with more force than if it be placed at the same distance from the larger body.

I think therefore that we may be well assured that if two similar bodies are connected together by a slender canal, and are overcharged, the fluid must escape faster from the smaller body than from an equal surface of the larger; but as the surface of the larger body is greatest, I do not know which body ought to lose most electricity in the same time; and indeed it seems impossible to determine positively from this theory which should, as it depends in great measure on the manner in which the air opposes the entrance of the electric fluid into it. Perhaps in some degrees of electrification the smaller body may lose most, and in others the larger.

Let now A C B (fig. 18.) be a conical point standing on any body D A B, C being the vertex of the cone; and let D A B be overcharged: I imagine that a particle of fluid placed close to the surface of the cone anywhere between *b* and C, must be repelled with at least as much, if not more, force than it would, if the part A *a b* B of the cone was taken away, and the part *a* C *b* connected to D A B by a slender canal; and consequently, from what has been said before, it seems reasonable to suppose that the waste of electricity from the end of the cone must be very great in proportion to its surface; though it does not appear from this reasoning whether the waste of electricity from the whole cone should be greater or less than from a cylinder of the same base and altitude.

All

All which has been here said relating to the flow-ing out of electricity from overcharged bodies, holds equally true with regard to the flowing in of electricity into undercharged bodies.

But a circumstance which I believe contributes as much as any thing to the quick discharge of electricity from points, is the swift current of air caused by them, and taken notice of by Mr. Wilson and Dr. Priestly (*vide* Priestly, p. 117 and 591); and which is produced in this manner.

If a globular body A B D is overcharged, the air close to it, all round its surface, is rendered over-charged, by the electric fluid, which flows into it from the body; it will therefore be repelled by the body; but as the air all round the body is repelled with the same force, it is in equilibrio, and has no tendency to fly off from it. If now the conical point A C B be made to stand out from the globe, as the fluid will escape much faster in proportion to the surface from the end of the point than from the rest of the body, the air close to it will be much more overcharged than that close to the rest of the body; it will therefore be repelled with much more force; and consequently a current of air will flow along the sides of the cone, from B towards C; by which means there is a continual supply of fresh air, not much overcharged, brought in contact with the point; whereas otherwise the air adjoining to it would be so much overcharged, that the electricity would have but little disposition to flow from the point into it.

The same current of air is produced in a less degree, without the help of the point, if the body, instead of being globular, is oblong or flat, or has

knobs

knobs on it, or is otherwise formed in such manner as to make the electricity escape faster from some parts of it than the rest.

In like manner, if the body A B D be undercharged, the air adjoining to it will also be undercharged, and will therefore be repelled by it; but as the air close to the end of the point will be more undercharged than that close to the rest of the body, it will be repelled with much more force; which will cause exactly the same current of air, flowing the same way, as if the body was overcharged; and consequently the velocity with which the electric fluid flows into the body, will be very much increased. I believe indeed that it may be laid down as a constant rule, that the faster the electric fluid escapes from any body when overcharged, the faster will it run into that body when undercharged.

Points are not the only bodies which cause a quick discharge of electricity; in particular, it escapes very fast from the ends of long slender cylinders; and a swift current of air is caused to flow from the middle of the cylinder towards the end: this will easily appear by considering that the redundant fluid is collected in much greater quantity near the ends of the cylinders than near the middle. The same thing may be said, but I believe in a less degree, of the edges of thin plates.

What has been just said concerning the current of air, serves to explain the reason of the revolving motion of Dr. Hamilton's and Mr. Kinnersley's bent pointed wires, vide Phil. Transf. vol. LI, p. 905, and vol. LIII, p. 86; also Priestly, p. 429: for the same repulsion which impels the air from the thick part of the

wire towards the point, tends to impel the wire in the contrary direction.

It is well known, that if a body B is positively electrified, and another body A, communicating with the ground, be then brought near it, the electric fluid will escape faster from B, at that part of it which is turned towards A, than before. This is plainly conformable to theory; for as A is thereby rendered undercharged, B will in its turn be made more overcharged, in that part of it which is turned towards A, than it was before. But it is also well known that the fluid will escape faster from B, if A be pointed, than if it be blunt; though B will be less overcharged in this case than in the other; for the broader the surface of A, which is turned towards B, the more effect will it have in increasing the overcharge of B. The cause of this phænomenon is as follows:

If A is pointed, and the pointed end turned towards B, the air close to the point will be very much undercharged, and therefore will be strongly repelled by A, and attracted by B, which will cause a swift current of air to flow from it towards B; by which means a constant supply of undercharged air will be brought in contact with B, which will accelerate the discharge of electricity from it in a very great degree: and moreover, the more pointed A is, the swifter will be this current. If, on the other hand, that end of A which is turned towards B, is so blunt, that the electricity is not disposed to run into A faster than it is to run out of B, the air adjoining to B may be as much overcharged as that adjoining to A is undercharged; and therefore may by the joint repulsion

of

of B and attraction of A, be impelled from B to A, with as much or more force than the air adjoining to A is impelled in the contrary direction; so that what little current of air there is may flow in the contrary direction.

It is easy applying what has been here said to the case in which B is negatively electrified.

§ 5. In the paper of Mr. Canton's, quoted in the second section, and in a paper of Dr. Franklin's (Phil. Transf. 1755, p. 300, and Franklin's letters p. 155.) are some remarkable experiments, shewing that when an overcharged body is brought near another body, some fluid is driven to the further end of this body, and also some driven out of it, if it is not insulated. The experiments are all strictly conformable to the 11th, 12th, and 13th propositions: but it is needless to point out the agreement, as the explanation given by the authors does it sufficiently.

§ 6. On the Leyden vial.

The shock produced by the Leyden vial seems owing only to the great quantity of redundant fluid collected on its positive side, and the great deficiency on its negative side; so that if a conductor was prepared of so great a size, as to be able to receive as much additional fluid by the same degree of electrification as the positive side of a Leyden vial, and was positively electrified in the same degree as the vial, I do not doubt but what as great a shock would be produced by making a communication between this conductor and the ground, as between the two surfaces of the

Leyden

Leyden vial, fuppofing both communications to be made by canals of the fame length and fame kind.

It appears plainly from the experiments which have been made on this fubject, that the electric fluid is not able to pafs through the glafs; but yet it feems as if it was able to penetrate without much difficulty to a certain fmall depth, perhaps I might fay an imperceptible depth within the glafs; as Dr. Franklin's analyfis of the Leyden vial fhews that its electricity is contained chiefly in the glafs itfelf, and that the coating is not greatly over or undercharged.

It is well known that glafs is not the only fubftance which can be charged in the manner of the Leyden vial; but that the fame effect may be produced by any other body, which will not fuffer the electricity to pafs through it.

* Hence the phænomena of the vial feem eafily explicable by means of the 22d propofition. For let A C G M, fig. 20, reprefent a flat plate of glafs or any other fubftance which will not fuffer the electric fluid to pafs through it, feen edgeways; and let B *b d* D, and E *e f* F, or B *d* and E *f*, as I fhall call them for fhortnefs, be two plates of conducting matter of the fame fize, placed in contact with the glafs oppofite to each other; and let B *d* be pofitively electrified; and let E *f* communicate with the ground; and let the fluid be fuppofed either

* The following explication is ftrictly applicable only to that fort of Leyden vial, which confifts of a flat plate of glafs or other matter. It is evident, however, that the refult muft be nearly of the fame kind, though the glafs is made into the fhape of a bottle as ufual, or into any other form: but I propofe to confider thofe fort of Leyden vials more particularly in a future paper.

able

able to enter a little way into the glafs, but not to pafs through it, or unable to enter it at all; and if it is able to enter a little way into it, let $b \beta \delta d$, or $b \delta$, as I fhall call it, reprefent that part of the glafs into which the fluid can enter from the plate Bd, and $e \varphi$, that which the fluid from Ef can enter. By the abovementioned propofition, if $b e$, the thicknefs of the glafs, is very fmall in refpect of $b d$, the diameter of the plates, the quantity of redundant fluid forced into the fpace Bd, or Bδ, (that is, into the plate Bd, if the fluid is unable to penetrate at all into the glafs, or into the plate Bd, and the fpace $b \delta$ together, if the fluid is able to penetrate into the glafs) will be many times greater than what would be forced into it by the fame degree of electrification if it had been placed by itfelf; and the quantity of fluid driven out of Eφ, will be nearly equal to the redundant fluid in Bδ.

If a communication be now made between Bδ and Eφ, by the canal N R S, the redundant fluid will run from Bδ to Eφ; and if in its way it paffes through the body of any animal, it will by the rapidity of its motion produce in it that fenfation called a fhock.

It appears from the 26th propofition, that if a body of any fize was electrified in the fame degree as the plate Bd, and a communication was made between that body and the ground, by a canal of the fame length, breadth and thicknefs as N R S; that then the fluid in that canal would be impelled with the fame force as that in N R S, fuppofing the fluid in both canals to be incompreffible; and confequently, as the quantity of fluid to be moved,

and the refiftance to its motion is the fame in both canals, the fluid fhould move with the fame rapidity in both : and I fee no reafon to think that the cafe will be different, if the communication is made by canals of real fluid.

Therefore what was faid in the beginning of this fection, namely, that as great a fhock would be produced by making a communication between the conductor and the ground, as between the two fides of the Leyden vial, by canals of the fame length and fame kind, feems a neceffary confequence of this theory; as the quantity of fluid which paffes through the canal is, by the fuppofition, the fame in both; and there is the greateft reafon to think, that the rapidity with which it paffes will be nearly if not quite the fame in both. I hope foon to be able to fay whether this agrees with experiment as well as theory.

It may be worth obferving, that the longer the canal N R S is, by which the communication is made, the lefs will be the rapidity with which the fluid moves along it; for the longer the canal is, the greater is the refiftance to the motion of the fluid in it; whereas the force with which the whole quantity of fluid in it is impelled, is the fame whatever be the length of the canal. Accordingly, it is found in melting fmall wires, by directing a fhock through them, that the longer the wire the greater charge it requires to melt it.

As the fluid in B δ, is attracted with great force by the redundant matter in E φ, it is plain that if the fluid is able to penetrate at all into the glafs, great part of the redundant fluid will be lodged in

$b \delta,$

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$b\delta$, and in like manner there will be a great defi-
cience of fluid in $e\varphi$. But in order to form fome
eftimate of the proportion of the redundant fluid,
which will be lodged in $b\delta$, let the communication
between Ef and the ground be taken away, as well
as that by which Bd is electrified; and let fo much
fluid be taken from $B\delta$, as to make the redundant
fluid therein equal to the deficient fluid in $E\varphi$. If
we fuppofe that all the redundant fluid is collected
in $b\delta$, and all the deficient in $e\varphi$, fo as to leave
Bd and Ef faturated; then, if the electric repul-
fion is inverfely as the fquare of the diftance, a
particle of fluid placed anywhere in the plane bd,
except near the extremities b and d, will be at-
tracted with very near as much force by the redun-
dant matter in $e\varphi$, as it is repelled by the redun-
dant fluid in $b\delta$; but if the repulfion is inverfely,
as fome higher power than the fquare, it will be
repelled with much more force by $b\delta$, than it is.
attracted by $e\varphi$, provided the depth $b\beta$ is very
fmall in refpect of the thicknefs of the glafs; and
if the repulfion is inverfely, as fome lower power
than the fquare, it will be attracted with much
more force by $e\varphi$, than it is repelled by $b\delta$. Hence
it follows, that if the depth to which the fluid can
penetrate is very fmall in refpect of the thicknefs
of the glafs, but yet is fuch that the quantity of
fluid naturally contained in $b\delta$, or $e\varphi$, is confi-
derably more than the redundant fluid in $B\delta$; then,
if the repulfion is inverfely as the fquare of the
diftance, almoft all the redundant fluid will be
collected in $b\delta$, leaving the plate Bd not very
much overcharged; and in like manner Ef will be

not

not very much undercharged: if the repulsion is inversely as some higher power than the square, B *d* will be very much overcharged, and E *f* very much undercharged: and if the repulsion is inversely, as some lower power than the square, B *d* will be very much undercharged, and E *f* very much overcharged.

Suppose, now, the plate B *d* to be separated from the plate of glass, still keeping it parallel thereto, and opposite to the same part of it that it before was applied to; and let the repulsion of the particles be inversely, as some higher power of the distance than the square. When the plate is in contact with the glass, the repulsion of the redundant fluid in that plate, on a particle in the plane *b d*, *id est*, the inner surface of the plate, must be equal to the excess of the repulsion of the redundant fluid in *b δ* on it, above the attraction of E φ on it; therefore, when the plate B *d* is removed ever so small a distance from the glass, the repulsion of the redundant fluid in the plate, on a particle in the inner surface of that plate, will be greater than the excess of the repulsion of *b δ* on it, above the attraction of E φ; for the repulsion of *b δ* will be much more diminished by the removal, than the attraction of E φ: consequently, some fluid will fly from the plate to the glass, in the form of sparks: so that the plate will not be so much overcharged when removed from the glass, as it was when in contact with it. I should imagine, however, that it would still be considerably overcharged.

If one part of the plate is separated from the glass before the rest, as must necessarily be the case, if it consists of bending materials, I should guess it would

be

be at leaft as much, if not more, overcharged, when feparated, as if it is feparated all at once.

In like manner, it fhould feem that the plate E*f* will be confiderably undercharged, when feparated from the glafs, but not fo much fo as when in contact with it.

From the fame kind of reafoning I conclude, that if the repulfion is inverfely, as fome lower power of the diftance than the fquare, the plate B*d* will be confiderably undercharged, and E*f* confiderably overcharged, when feparated from the glafs, but not in fo great a degree as when they are in contact with it.

§ 7. There is an experiment of Mr. Wilke and Æpinus, related by Dr. Prieftly, p. 258. called by them, electrifying a plate of air: it confifted in placing two large boards of wood, covered with tin plates, parallel to each other, and at fome inches afunder. If a communication was made between one of thefe and the ground, and the other was pofitively electrified, the former was undercharged; the boards ftrongly attracted each other; and, on making a communication between them, a fhock was felt like that of the Leyden vial.

I am uncertain whether in this experiment the air contained between the two boards is very much overcharged on one fide, and very much undercharged on the other, as is the cafe with the plate of glafs in the Leyden vial; or whether the cafe is, that the redundant or deficient fluid is lodged only in the two boards, and that the air between them ferves only to prevent the electricity from running from one

board to the other: but whichever of thefe is the cafe, the experiment is equally conformable to the theory.

It muft be obferved, that a particle of fluid placed between the two plates is drawn towards the under-charged plate, with a force exceeding that with which it would be repelled from the overcharged plate, if it was electrified with the fame force, the other plate being taken away, nearly in the ratio of twice the quantity of redundant fluid actually contained in the plate, to that which it would contain, if electrified with the fame force by itfelf; fo that, unlefs the plate is very weakly electrified, or their diftance is very confiderable, the fluid will be apt to fly from one to the other, in the form of fparks.

§ 8. Whenever any conducting body as A, com-municating with the ground, is brought fufficiently near an overcharged body B, the electric fluid is apt to fly through the air from B to A, in the form of a fpark: the way by which this is brought about feems to be this. The fluid placed anywhere be-tween the two bodies, is repelled from B towards A, and will confequently move flowly through the air from one to the other: now it feems as if this mo-tion increafed the elafticity of the air, and made it rarer: this will enable the fluid to flow in a fwifter current, which will ftill further increafe the elafti-city of the air, till at laft it is fo much rarified, as to form very little oppofition to the motion of the elec-tric fluid, upon which it flies in an uninterrupted mafs from one body to the other.

In

In the same manner may the electric fluid pass from one body to another, in the form of a spark, if the first body communicates with the ground, and the other body is negatively electrified, or in any other case in which one body is strongly disposed to part with its electricity to the air, and the other is strongly disposed to receive it.

In like manner, when the electric fluid is made to pass through water, in the form of a spark, as in Signor Beccaria's * and Mr. Lane's † experiments, I imagine that the water, by the rapid motion of the electric fluid through it, is turned into an elastic fluid, and so much rarified as to make very little opposition to its motion: and when stones are burst or thrown out from buildings struck by lightning, in all probability that effect is caused by the moisture in the stone, or some of the stone itself, being turned into an elastic fluid.

It appears plainly, from the sudden rising of the water in Mr. Kinnersley's electrical air thermometer ‡, that when the electric fluid passes through the air, in the form of a spark, the air in its passage is either very much rarified, or intirely displaced: and the bursting of the glass vessels, in Beccaria's and Lane's experiments, shews that the same thing happens with regard to the water, when the electric fluid passes through it in the form of a spark. Now, I see no means by which the displacing of the air or

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* Elettricismo artificiale e naturale, p. 110. Priestly, p. 209.

† Phil. Transf. 1767, p. 451.

‡ Phil. Transf. 1763, p. 84. Priestly, p. 216.

water

water can be brought about, but by fuppoſing its elaſticity to be increaſed, by the motion of the electric fluid through it, unleſs you fuppoſe it to be actually puſhed aſide, by the force with which the electric fluid endeavours to iſſue from the overcharged body: but I can by no means think, that the force with which the fluid endeavours to iſſue, in the ordinary caſes in which electric ſparks are produced, is ſufficient to overcome the preſſure of the atmoſphere, much leſs that it is ſufficient to burſt the glaſs veſſels in Beccaria's and Lane's experiments.

The truth of this is confirmed by Prop. XVI. For, let an undercharged body be brought near to, and oppoſite to the end of a long cylindrical body communicating with the ground, by that propoſition the preſſure of the electric fluid againſt the baſe of the cylinder is ſcarcely greater than the force with which the two bodies attract each other, provided that no part of the cylinder is undercharged; which is very unlikely to be the caſe, if the electric repulſion is inverſely as the ſquare of the diſtance, as I have great reaſon to believe it is; and, conſequently, if the ſpark was produced, by the air being puſhed aſide by the force with which the fluid endeavours to iſſue from the cylinder, no ſparks ſhould be produced, unleſs the electricity was ſo ſtrong, that the force with which the bodies attracted each other was as great as the preſſure of the atmoſphere againſt the baſe of the cylinder: whereas it is well known, that a ſpark may be produced, when the force, with which the bodies attract, is very trifling in reſpect of that.

One

One may frequently obferve, in difcharging a Leyden vial, that if the two knobs are approached together very flowly, a hiffing noife will be perceived before the fpark ; which fhews, that the fluid begins to flow from one knob to the other, before it paffes in the form of a fpark ; and therefore ferves to confirm the truth of the opinion, that the fpark is brought about in the gradual manner here defcribed.

A N